３体問題とのかかわり，最近の話

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以下の３つは，ここ数年，私のかかわった３体問題に関する応用数学論文のタイトルとアブストラクトです．この順番で最初の話をメインに説明したいと思います.

Choreographic Three Bodies on the Lemniscate

We show that choreographic three bodies $\{\mathbf{x(t)}, \mathbf{x(t+T/3)}, \mathbf{x(t-T/3)}\}$ of period T on the lemniscate, $\mathbf{x(t)} = (\hat{\mathbf{x}} + \hat{\mathbf{y}}\mathrm{cn(t)})\mathrm{sn(t)}/(1+\mathrm{cn}^2\mathrm{(t)})$ parameterized by the Jacobi's elliptic functions sn and cn with modulus $k^2 = (2+\sqrt{3})/4$, conserve the center of mass and the angular momentum, where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the orthogonal unit vectors defining the plane of the motion. They also conserve the moment of inertia, the kinetic energy, the sum of square of the curvature, the product of distance and the sum of square of distance between bodies. We find that they satisfy the equation of motion under the potential energy $\sum_{i<j}(1/2\ln r_{ij} - \sqrt{3}/24r_{ij}^2)$ or $\sum_{i<j} 1/2\ln r_{ij} - \sum_i \sqrt{3}/8r_i^2$, where r_{ij} the distance between the body i and j and r_i the distance from the origin. The first term of the potential energies are both the Newton's gravity in two dimension but the second term is the mutual repulsive force or an repulsive force from the origin, respectively.

Evolution of the Moment of Inertia of Figure-Eight Choreography

We investigate three-body motion in three dimensions under the interaction potential proportional to r^α ($\alpha \neq 0$) or $\log r$, where r represents the mutual distance between bodies, with the following conditions: (I) the moment of inertia is non-zero constant, (II) the angular momentum is zero, and (III) one body is on the centre of mass at an instant.

We prove that the motion which satisfies conditions (I)–(III) with equal masses for $\alpha \neq -2, 2, 4$ is impossible. And motions which satisfy the same conditions for $\alpha = 2, 4$ are solved explicitly. Shapes of these orbits are not figure-eight and these motions have collision. Therefore the moment of inertia for figure-eight choreography for $\alpha \neq -2$ is proved to be inconstant along the orbit.

We also prove that the motion which satisfies conditions (I)–(III) with general masses under the Newtonian potential $\alpha = -1$ is impossible.

Three-body problem and sum rules for the associated Legendre polynomials

The three-body angular basis has been used to produce two infinite series of identities for the associated Legendre polynomials which are mostly known as two-body objects. The coefficients that are involved in the new sum rules are given in terms of the Clebsh-Gordan coefficients.