Precision spectroscopy of atomic hydrogen and the proton charge radius puzzle

N. Kolachevsky
Why proton charge radius is so important?

- needed for accurate Rydberg constant determination
Hydrogen atomic levels

Schrödinger (atomic units):

\[ E_{\text{Bohr}} = -\frac{1}{n^2} \]

QED:

\[ E_{\text{QED}} = -\frac{1}{n^2} + \frac{3(2j + 1) - 8n}{4(2j + 1)n^4} \alpha^2 + \ldots \]

\[ = -\frac{1}{n^2} + a_2 \alpha^2 + a_4 \alpha^4 + a_{50} \alpha^5 + a_{51} \alpha^5 \ln(\alpha^{-2}) + \ldots \]

recoil:

\[ + \frac{m_r}{2(M + m_e)} [f(n, j) - 1]^2 + \frac{(Z \alpha)^4 m_r^2}{2n^3 M^2} \left[ \frac{1}{j + 1/2} - \frac{1}{l + 1/2} \right] (1 - \delta_{l0}) \]

nuclear size correction:

\[ + \frac{2\pi (Z \alpha)}{3} \langle r_p^2 \rangle |\psi(0)|^2 \]
Fundamental constants

\[ E_n = R_\infty F \left( \alpha, \frac{m_e}{M}, \langle r_p^2 \rangle \right) \]

Relative uncertainties:

\[ \alpha: \quad 3 \times 10^{-10} \]
\[ \frac{m_e}{M}: \quad 4 \times 10^{-10} \]
\[ \langle r_p^2 \rangle: \quad 1\% \]

Do not contribute (small corrections!)

<table>
<thead>
<tr>
<th>( L_{1S} )</th>
<th>self-energy</th>
<th>vacuum pol.</th>
<th>( r_p )</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>e p</td>
<td>8 383 MHz</td>
<td>-215 MHz</td>
<td>1.253(50) MHz</td>
<td>8 172 MHz</td>
</tr>
</tbody>
</table>
Method 1: Electron scattering

\[ e^- \rightarrow p^+ \]

Measured value \[ \frac{d\sigma}{d\Omega} \]

Mott cross-section:

\[ \left. \frac{d\sigma}{d\Omega} \right|_{\text{point}} = \frac{\alpha^2}{4p_e^2 \sin^4 \frac{\theta}{2}} \left( \cos^2 \frac{\theta}{2} - \frac{q^2}{2m_p^2} \sin^2 \frac{\theta}{2} \right) \]

Non point-like proton:

\[ \frac{d\sigma}{d\Omega} = \left. \frac{d\sigma}{d\Omega} \right|_{\text{point}} |F(q^2)|^2 \]

For small \( q \)

\[ F(0) = 1 \]

impact parameter >> proton size
Form-factor and charge radius

Form-factor is the Fourier transformation of the charge distribution

\[ F(q^2) = \int \rho(x) e^{i\vec{q} \cdot \vec{x}} d^3x \]

Mean proton charge radius

\[ \rho(r) = \rho_0 e^{-r/r_0}, \quad r_0 \approx 0.8 \text{ fm} \]

Since proton possesses anomalous magnetic moment:

\[ \mu_p = \frac{(1+\kappa_p)e}{2m_p} \]

\[ \kappa_p = 1.79 \]
Electric and magnetic form-factors

$G_E = F_1 + \frac{\kappa q^2}{4m_p^2} F_2$

$G_M = F_1 + \kappa F_2$

$\langle r_p^2 \rangle = -6\hbar^2 \left. \frac{dG_E^2(q^2)}{dq^2} \right|_{Q=0}$

Derivative at $q=0$
e-p scattering results

\[ r_p = 0.895(18) \text{ fm} \quad (\sigma_r = 2\%) \]
Hydrogen spectroscopy

\[ E_n = R_\infty F\left(\alpha, \frac{m_e}{M}, \left\langle r_p^2 \right\rangle\right) \]

Two unknowns ⇒

Two equations necessary!

Hydrogen spectroscopy (Lamb shift):

\[ L_{1S}(r_p) = 8171.636(4) + 1.5645 \left\langle r_p^2 \right\rangle \text{ MHz} \]

\[ n=2 \text{ unknowns} \Rightarrow 2 \text{ transitions} \]

- Rydberg constant \( R_\infty \)
- Lamb shift \( L_{1S} \leftarrow r_p \)
Precision spectroscopy in H:

- ALL levels except 2S promptly decay

\[ \gamma(3S) \approx \gamma(3P) \approx \gamma(4P) \approx 10 \text{ MHz} \]

- Natural linewidth of Rydberg states reduces, but they become sensitive to electric fields (Stark effect)

\[ \Delta E_{DC} \approx n^7 \]

Scattered electric field at the level of 1V/m is hard to control!
Hydrogen spectroscopy: results

$r_p = 0.8768(69) \text{ fm}$

$(u_r = 0.8\%)$

Dominating uncertainty results from spectroscopy of higher excited states (not 2S)
Muonic hydrogen $\mu p$

$\mu^- p^+$

$m_\mu/m_e \approx 200$

<table>
<thead>
<tr>
<th></th>
<th>Hydrogen</th>
<th>Muonic hydrogen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bohr radius</td>
<td>$\hbar/mc\alpha$</td>
<td>50 pm</td>
</tr>
<tr>
<td>Lyman-$\alpha$</td>
<td>$3mc^2\alpha^2/8$</td>
<td>121 nm (10 eV)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L_{2S}$</th>
<th>self-energy</th>
<th>vacuum pol.</th>
<th>$r_p$</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>e p</td>
<td>1 085 MHz</td>
<td>-27 MHz</td>
<td>0.14 MHz</td>
<td>1 057 MHz</td>
</tr>
<tr>
<td>$\mu$ p</td>
<td>0.1 THz</td>
<td>-45 THz</td>
<td>0.93 THz</td>
<td>-49 THz</td>
</tr>
</tbody>
</table>
Spectroscopy of 2S-2P transition in $\mu p$

Transition line width 10 GHz
Goal – uncertainty of 1 GHz (1999)

$\mu^-$ stop in $H_2$ gas
$\Rightarrow \mu p^*$ atoms formed ($n \sim 14$)

99%: cascade to $\mu p(1S)$, emitting prompt $K_\alpha, K_\beta$ ...

1%: long-lived $\mu p(2S)$ atoms
lifetime $\tau_{2S} \approx 1 \mu s$ at 1 mbar $H_2$

Target (H$_2$)
Resonance line 2S-2P in $\mu p$

Water-line/laser wavelength: 300 MHz uncertainty

$\Delta \nu$ water-line to resonance: 200 kHz uncertainty

CODATA-06

$e-p$ scattering

H$_2$O calib.

Systematics: 300 MHz
Statistics: 700 MHz

Discrepancy:
$5.0 \sigma \leftrightarrow 75 \text{ GHz} \leftrightarrow \delta \nu / \nu = 1.5 \times 10^{-3}$

Proton radius 2010

\[ r_p = 0.84184 (36)_{\text{exp}}^{(56)}_{\text{theo}} \text{ fm} \]
Where the problem resides ?!

- Errors in QED calculations are excluded at such level of discrepancy

“New” physics?

proton polarizability aka. two-photon exchange

Seems to be the only contribution which might be able to solve the proton size puzzle by changing theory in $\mu_p$.

Keep in mind:
- Discrepancy: 0.31 meV
- Polarizability: 0.015(4) meV  20 times smaller!
Are

- e-p scattering experiments
- H spectroscopy results

flawless?
Hydrogen spectroscopy

\[
\begin{align*}
2S_{1/2} - 2P_{1/2} \\
2S_{1/2} - 2P_{3/2} \\
1S - 2S + 2S - 4S_{1/2} \\
1S - 2S + 2S - 4D_{5/2} \\
1S - 2S + 2S - 4P_{1/2} \\
1S - 2S + 2S - 4P_{3/2} \\
1S - 2S + 2S - 6S_{1/2} \\
1S - 2S + 2S - 6D_{5/2} \\
1S - 2S + 2S - 8S_{1/2} \\
1S - 2S + 2S - 8D_{3/2} \\
1S - 2S + 2S - 8D_{5/2} \\
1S - 2S + 2S - 12D_{3/2} \\
1S - 2S + 2S - 12D_{5/2} \\
1S - 2S + 1S - 3S_{1/2}
\end{align*}
\]

Proton charge radius (fm)

\[H_{\text{avg}} = 0.8779 \pm 0.0094 \text{ fm}\]

\[\mu_p = 0.84184 \pm 0.00067 \text{ fm}\]
Measurements of the 1S-2S transition

(f_{1S-2S} - 2466061431187080.5 \text{Hz} \) / Hz

June/July 1999  February 2003  May/June 2010

30 (44) Hz  1 (34) Hz

The Team

Hydrogen

Christian Parthey
Arthur Matveev
Janis Alnis
Axel Beyer
Nikolai Kolachevsky
Randolf Pohl
Thomas Udem
Theodor Häncsh

Frequency comb

Tobias Wilken
Birgitta Benhardt

Theory

Ulrich Jents
Brett Altschul

Mobile fountain clock

Michel Abgrall
Daniele Rovera
Christophe Salomon
Philippe Laurent

Optical fiber link

Katharina Predehl
Stefan Droste
Ronald Holzwarth
Harald Schnatz
Gesine Grosche
Thomas Legero
Stefan Weyers
The team 2009-

A. Beyer, Ph.D.

J. Alnis
Post. Doc.

T.W. Hänisch

A. Matveev
Post. Doc.

K. Khabarova
Post. Doc.

C. Partey
Ph. D.

Th. Udem

me
Setup

\[ \eta = \exp\left(-\varphi_{rms}^2\right) \text{ at 972 nm} \]

8-photon process \(\Rightarrow\) \(\eta_{eff} = \eta_{IR}^{64}\)

**Beat note diode - dye laser**

972 nm diode laser with a 20 cm long external resonator and intra-cavity EOM
FP1-FP2 Allan deviation

Thermal noise limit
Comb-comb comparison setup

Hydrogen maser → Distribution amplifier → Wenzel Chain \( \otimes 4 \otimes 5 \otimes 5 \) → PLL

Temperature stabilized

Stabilized optical fiber

Fiber Laser

1542 nm

1GHz

250 MHz system

100 MHz system
Comb-comb comparison: result

mean = -1.8+14.2 mHz

\[ \frac{2 \text{ mHz}}{200 \text{ THz}} = 7 \times 10^{-17} \]
The Hydrogen spectrometer

1S-2S spectrometer
Line Profile

average power = 6.54 V_{PD}

Lyman-\(\alpha\) count rate [cps]

frequency detuning @ 121nm [kHz]
Line Profile

![Graph showing Lyman-α count rate vs. frequency detuning at 121 nm (kHz). The graph includes data points for different delays: delay $\tau = 810\mu s$, delay $\tau = 1010\mu s$, delay $\tau = 1210\mu s$, and delay $\tau = 1410\mu s$. The average power is $6.54 V_{PD}$.](image-url)
Line Profile

\[ f_{H}^{(1S-2S)} - 246606102474849 \text{[Hz]} \]

excitation power [mW]

Lyman-\(\alpha\) count rate [cps]

Lyman-\(\alpha\) count rate [cps]

\[ \text{delay } \tau = 810 \mu \text{s} \]
Line Profile

Lyman-\(\alpha\) count rate [cps]

\(f(1S-2S) - 2466061102474849\) [Hz]

Delay \(\tau = 810\) \(\mu\)s

Delay = 1010
Mean = -25(10) Hz
\(\chi^2/\text{dof} = 1.4\)

Measurement run

Count rate vs. measurement run
## Uncertainty Budget

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Uncertainty [Hz]</th>
<th>Relative Uncertainty [$10^{-15}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>statistics</td>
<td>6.3</td>
<td>2.6</td>
</tr>
<tr>
<td>2nd order Doppler effect</td>
<td>5.1</td>
<td>2.0</td>
</tr>
<tr>
<td>line shape model</td>
<td>5.0</td>
<td>2.0</td>
</tr>
<tr>
<td>quadratic ac Stark shift (243 nm)</td>
<td>2.0</td>
<td>0.8</td>
</tr>
<tr>
<td>ac Stark shift, 486 nm quench light</td>
<td>2.0</td>
<td>0.8</td>
</tr>
<tr>
<td>hyperfine correction</td>
<td>1.7</td>
<td>0.69</td>
</tr>
<tr>
<td>dc Stark effect</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>ac Stark shift, 486 nm scattered light</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Zeeman shift</td>
<td>0.93</td>
<td>0.38</td>
</tr>
<tr>
<td>pressure shift</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>blackbody radiation shift</td>
<td>0.3</td>
<td>0.12</td>
</tr>
<tr>
<td>power modulation AOM chirp</td>
<td>0.3</td>
<td>0.11</td>
</tr>
<tr>
<td>rf discharge ac Stark shift</td>
<td>0.03</td>
<td>0.012</td>
</tr>
<tr>
<td>higher order modes</td>
<td>0.03</td>
<td>0.012</td>
</tr>
<tr>
<td>line pulling by $m_F = 0$ component</td>
<td>0.004</td>
<td>0.0016</td>
</tr>
<tr>
<td>recoil shift</td>
<td>0.009</td>
<td>0.0036</td>
</tr>
<tr>
<td>FOM</td>
<td>2.0</td>
<td>0.81</td>
</tr>
<tr>
<td>gravitational red shift</td>
<td>0.04</td>
<td>0.077</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10.4</strong></td>
<td><strong>4.2</strong></td>
</tr>
</tbody>
</table>
## Uncertainty Budget

<table>
<thead>
<tr>
<th></th>
<th>Uncertainty [Hz]</th>
<th>Relative Uncertainty [$10^{-15}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>statistics</td>
<td>6.3</td>
<td>2.6</td>
</tr>
<tr>
<td>2nd order Doppler effect</td>
<td>5.1</td>
<td>2.0</td>
</tr>
<tr>
<td>line shape model</td>
<td>5.0</td>
<td>2.0</td>
</tr>
<tr>
<td>quadratic ac Stark shift (243 nm)</td>
<td>2.0</td>
<td>0.8</td>
</tr>
<tr>
<td>ac Stark shift, 486 nm quench light</td>
<td>2.0</td>
<td>0.8</td>
</tr>
<tr>
<td>hyperfine correction</td>
<td>1.7</td>
<td>0.69</td>
</tr>
<tr>
<td>dc Stark effect</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>ac Stark shift, 486 nm scattered light</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Zeeman shift</td>
<td>0.93</td>
<td>0.38</td>
</tr>
<tr>
<td>pressure shift</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>blackbody radiation shift</td>
<td>0.3</td>
<td>0.12</td>
</tr>
<tr>
<td>power modulation AOM chirp</td>
<td>0.3</td>
<td>0.11</td>
</tr>
<tr>
<td>rf discharge ac Stark shift</td>
<td>0.03</td>
<td>0.012</td>
</tr>
<tr>
<td>higher order modes</td>
<td>0.03</td>
<td>0.012</td>
</tr>
<tr>
<td>line pulling by $m_F = 0$ component</td>
<td>0.004</td>
<td>0.0016</td>
</tr>
<tr>
<td>recoil shift</td>
<td>0.009</td>
<td>0.0036</td>
</tr>
<tr>
<td>FOM</td>
<td>2.0</td>
<td>0.81</td>
</tr>
<tr>
<td>gravitational red shift</td>
<td>0.04</td>
<td>0.077</td>
</tr>
<tr>
<td>total</td>
<td>10.4</td>
<td>4.2</td>
</tr>
</tbody>
</table>
Second Order Doppler Effect

\[ \Delta f_{dp} = -\frac{v^2}{(2c^2)} \cdot f_{1S - 2S} \]
Result

![Graph showing data points and error bars for different delays. The data points are labeled with their respective delay times, and the error bars represent the uncertainty in the measurements. The graph plots the quantity \( f_{1S-2S} - 24660614131870000 \) Hz vs. delay \( \tau \) in microseconds. The data points are marked with black circles, and a red circle highlights the mean value.](image-url)
Zeeman Shift

$$\Delta f(m_F = \pm 1) = \pm B \cdot 36 \text{ Hz/G}$$

$$B = 5G$$

Differential measurement:
30 winding    100 windings

$$I \rightarrow \pm I$$
Zeeman Shift

\[ \langle \Delta f \rangle_{\tau_{5,6}} = -4.8(5.8) \text{ Hz} \]

\[ \tau_5 = 810...1010 \mu s \]
\[ \tau_6 = 1010...1210 \mu s \]

5.8 Hz / 6.25 = 0.93 Hz \quad (0.38 \times 10^{-15})
Result

\[
\frac{f_{1S-2S} - 2.466601431870000\text{Hz}}{\text{Hz}}
\]

June/July 1999  February 2003  May/June 2010

109 (44) Hz  80 (34) Hz  35 (10) Hz

C.G. Parthey et al., PRL 107, 203001 (2011)
2 466 061 413 187 035(10) Hz

4.2 \times 10^{-15}
Optical Spectroscopy of Hydrogen

187 035(10) Hz

4.2 × 10^{-15}

C.G. Parthey et al., PRL 107, 203001 (2011)
gravitational red shift

\[ \frac{\Delta f}{f} = \frac{\Delta hg}{c^2} = 4.4 \times 10^{-14} \]
New detector

Faraday integrating sphere

UV photodiode
New Detector

delay = 1010\mu s

\(f_{1S-2S} - 246606141318700\ Hz/Hz\)
Result - Link Measurement

\[ (f_{1S-2S} - 24660614311870000Hz) / Hz \]

- 109 (44) Hz
- 80 (34) Hz
- 35 (10) Hz
- 18 (11) Hz

07/1999 - 11/2010
Uncertainly in the 1S-2S transition cannot solve the proton charge radius puzzle

2 466 061 413 187 035 (10) Hz

4.2 \times 10^{-15}
It is desirable to measure independently other transitions in H with higher accuracy.

We already use a laser to excite 2S-4P (486 nm) transition in a cold atomic beam of H.
2S-4P transition measurements (Yale’95)
2S-4P spectroscopy at MPQ

Cold beam of metastable atoms (4 K)

Optically populated only one hyperfine sublevel 2S (F=0)

Velocity selective detection, typical velocities down to 100 m/s

Frequency measurement is reliable
Natural line width of the 4P level

13 MHz
2S-4P experimental setup

- **Detection Region**: Channeltron
- **Excitation Region**: Faraday cage
- **486 nm**: Laser wavelength
- **H(2S)**: Hyperfine transition
Antiparallel Beams

Active fiber-based retroreflector:

\[ \tan(\theta) \approx \theta = \frac{\Delta x}{f} \]

\[ \Delta x \approx 3\mu m \]

\[ \Rightarrow \phi_{1/2} \approx 5 \cdot 10^{-5}\text{rad} \]

\[ \phi_{\text{FWHM, exp}} \approx 1.2 \cdot 10^{-4}\text{rad} \]
Inside the chamber
Gaussian beam

Assembly of spherical lenses

Aspherical lens
Data evaluation

Yale 1995

Winter 2012

Summer 2011

Spring 2011

2S - 4P frequency - 6.16520153e14Hz [kHz]
Data scattering (polarization dependent?)

\[
(f_{abs}^{2S-4P} - \text{offset}) = 103 \pm 1.8 \text{ kHz}
\]

\[
\chi^2/\text{dof} = 1.6
\]
Cross damping: coherent and incoherent parts

\[
\vec{d}(t) = \vec{d}_1 e^{i\omega_1 t - \Gamma_1/2t} + \vec{d}_2 e^{i\omega_2 t - \Gamma_2/2t + i\varphi}
\]

\[
\left|\tilde{d}(\omega)\right|^2 = \left|\frac{\vec{d}_1}{i(\omega - \omega_1) + \Gamma_1/2} + \frac{\vec{d}_2 e^{i\varphi}}{i(\omega - \omega_2) + \Gamma_2/2}\right|^2
\]

\[
= \frac{|\vec{d}_1|^2}{(\omega - \omega_1)^2 + (\Gamma_1/2)^2} + \frac{|\vec{d}_2|^2}{(\omega - \omega_2)^2 + (\Gamma_2/2)^2}
\]

\text{incoherent}

\[
+ \vec{d}_1 \cdot \vec{d}_2 \frac{\cos(\varphi) \left[ \Gamma_1 \Gamma_2/2 + 2(\omega - \omega_1)(\omega - \omega_2) \right] + \sin(\varphi) \left[ (\omega - \omega_1) \Gamma_2 - (\omega - \omega_2) \Gamma_1 \right]}{[(\omega - \omega_1)^2 + (\Gamma_1/2)^2][[(\omega - \omega_2)^2 + (\Gamma_2/2)^2]}
\]

\text{coherent}
Cross damping problem in the 2S-4P experiment

Simple rule: if one wants to split the line by the factor of $N$, the perturbing line should be further away as $N\gamma$

Analysis of systematic effects is still in progress
Thank you for attention!