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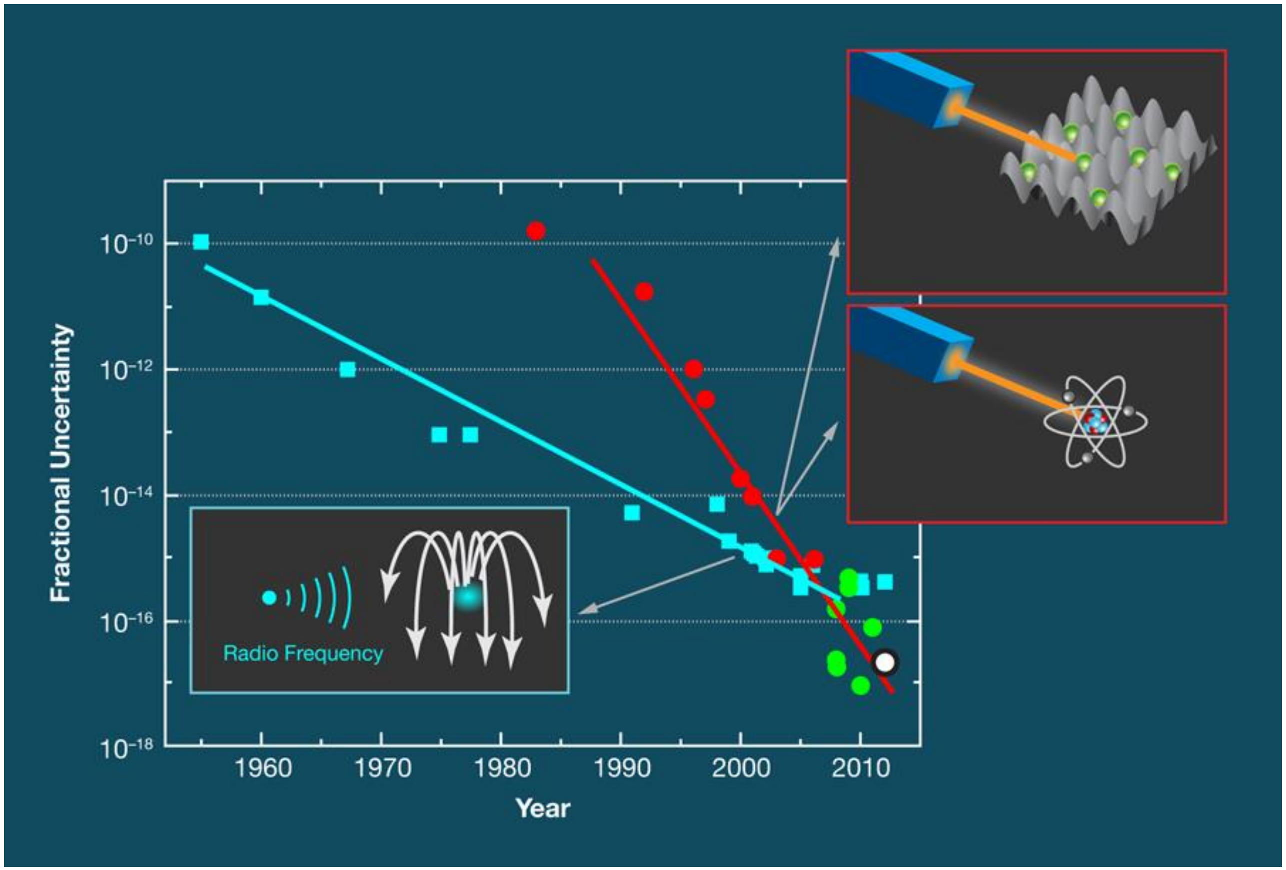
Lecture 1

- Frequency and time as most accurately measured quantities in physics.
- Clocks: from 17th century till today. Mechanical, radiofrequency, microwave and optical oscillators.
- Accuracy and stability. Phase and amplitude modulation, their mathematical representation and power spectrum.

From all known physical quantities, frequency can be measured with the highest accuracy.



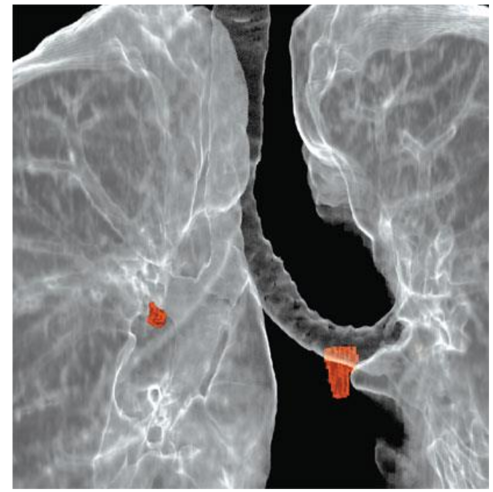
Today's best optical clock fractional uncertainty

$<10^{-17}$!

To measure a physical quantity with high accuracy, it is necessary to covert this quantity in frequency.

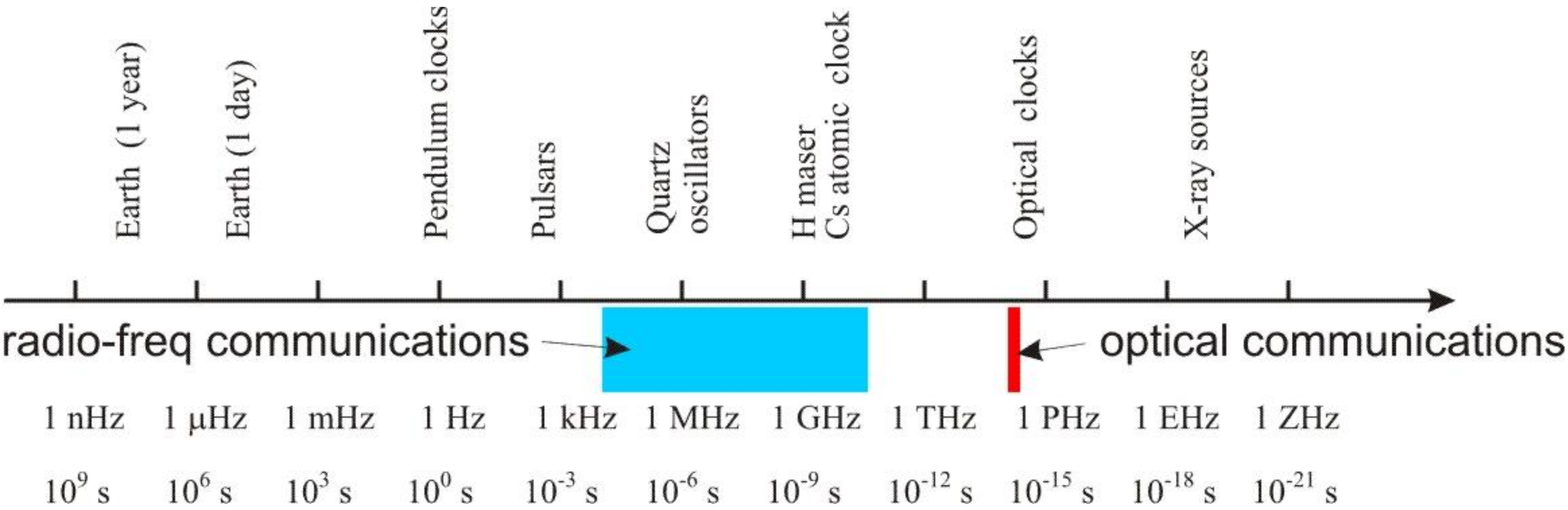
Road radar
velocity → frequency

Josephson effect
voltage → frequency



Medical tomograph image
local water density → frequency

Different oscillators and characteristic frequencies and time scales



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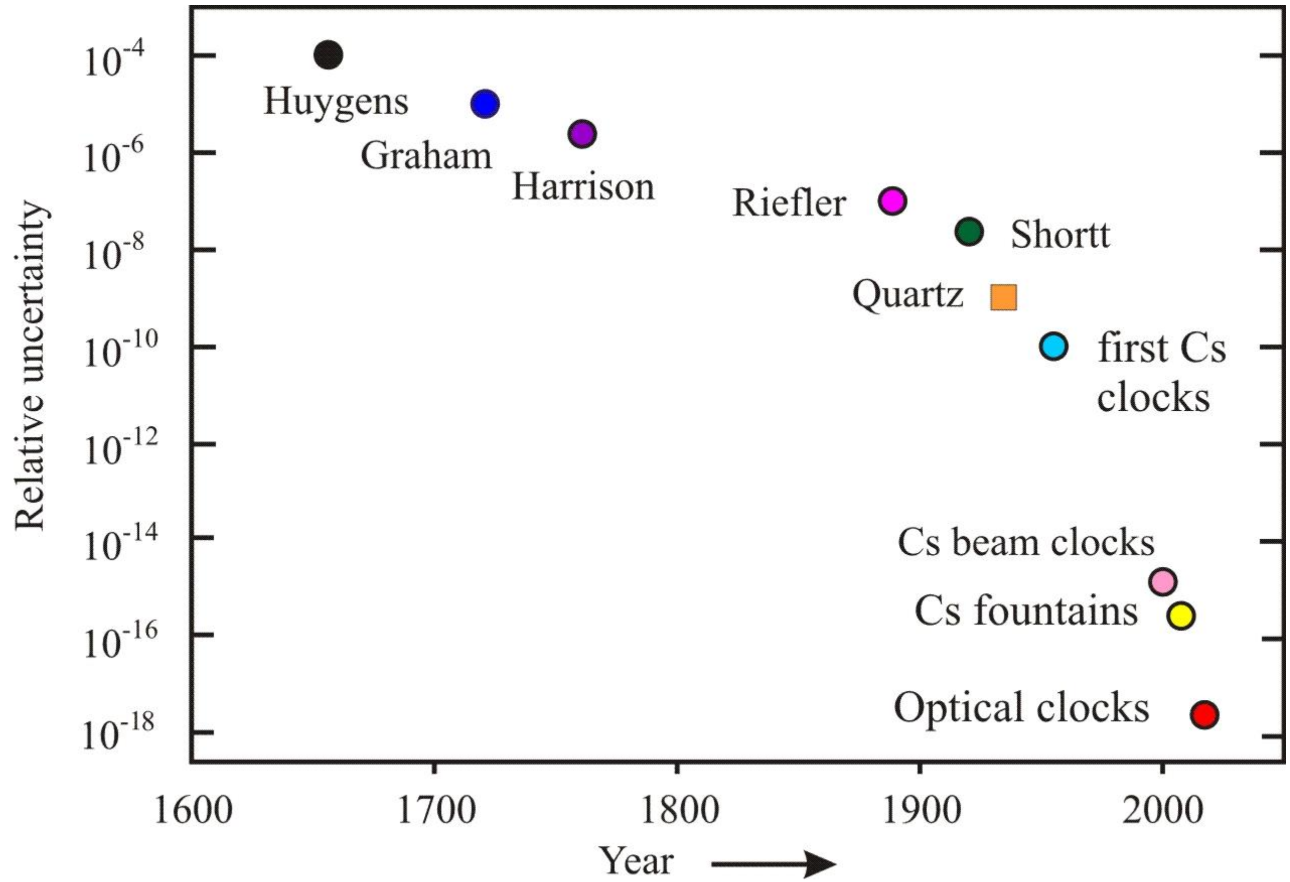
Progress of clock accuracy over last centuries



Mechanical clocks

First tower clocks accuracy: 15 min a day

$\sim 10^{-2}$

Best pendulum Shortt Clocks of 20th century

$\sim 10^{-8}$

Quartz crystalline oscillator

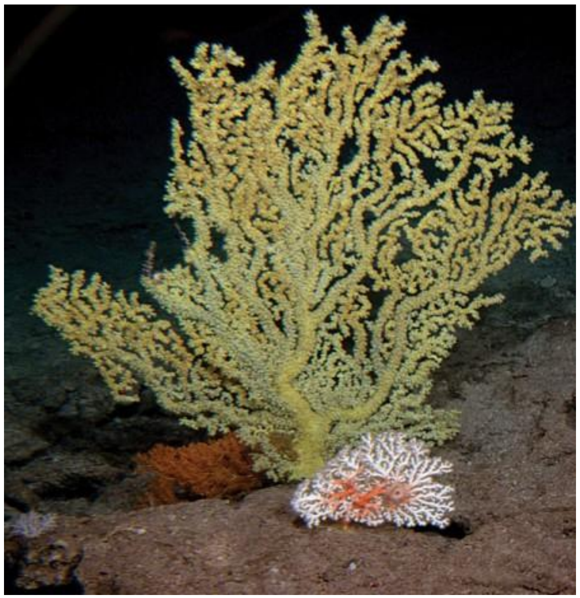
Typical instability in 1 day

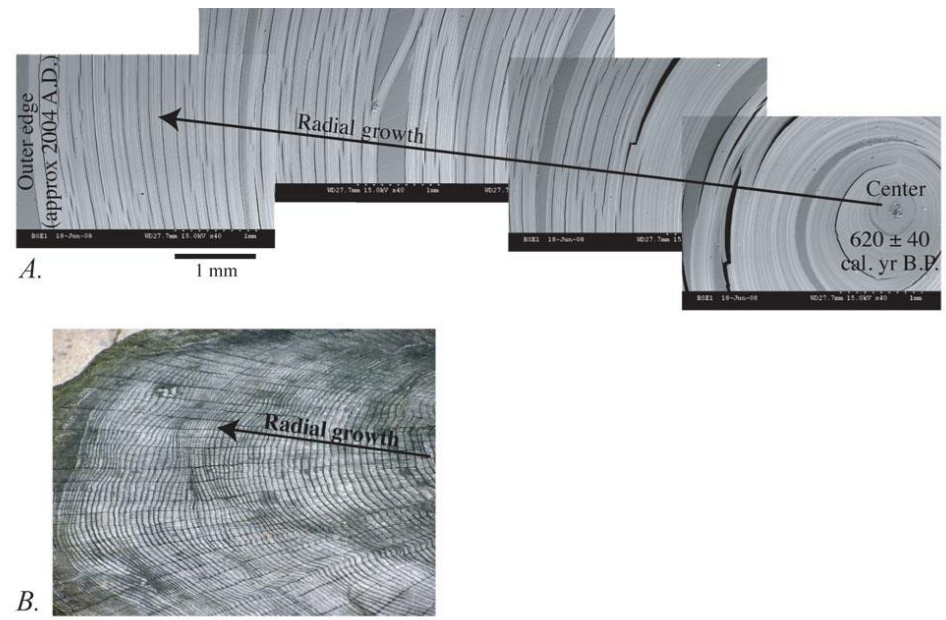
$\sim 10^{-8}$

Allowed to detect Earth deceleration!

Study of coral growth

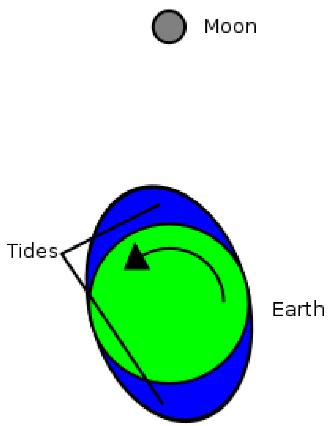
Earth rotation decelerates in time!

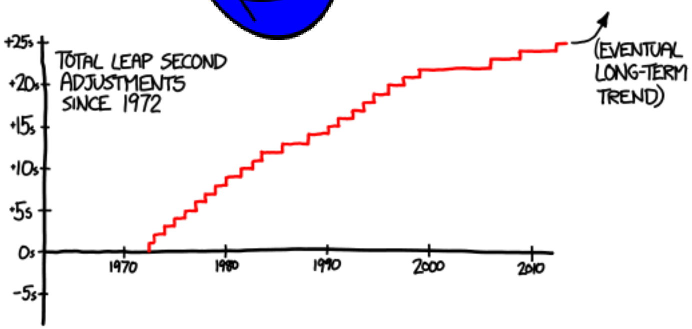


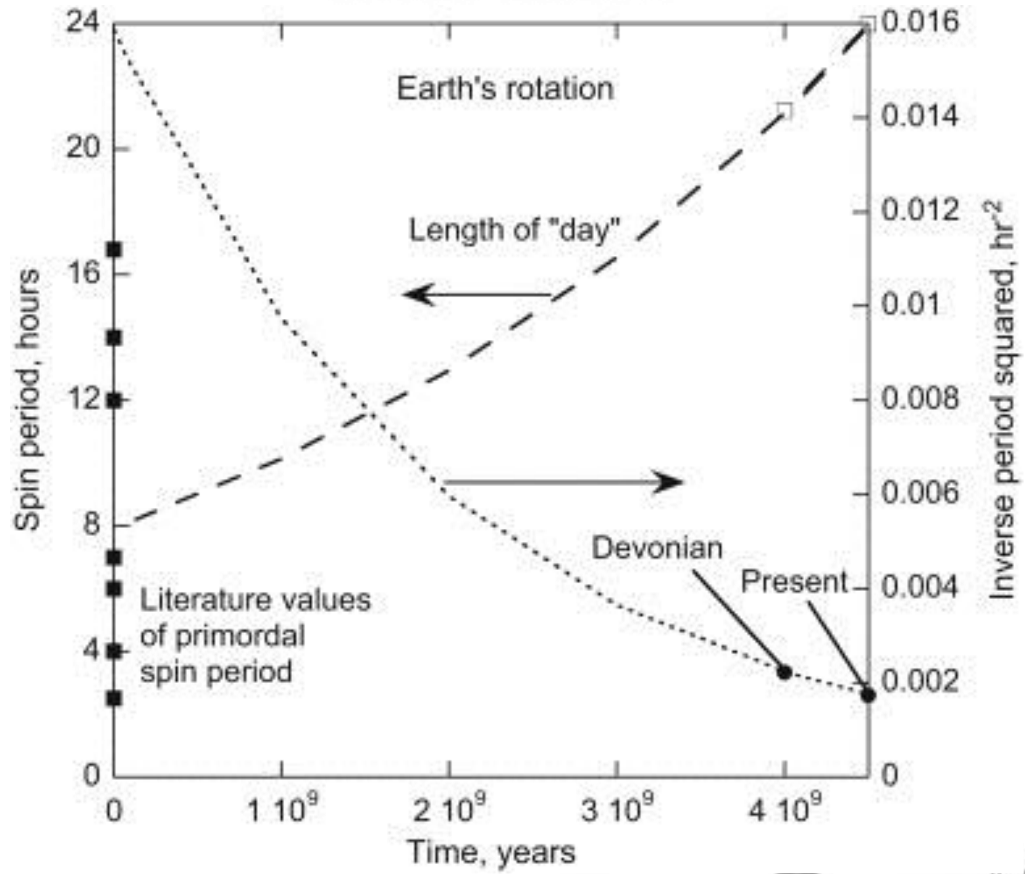


Tidal effects

1 billion years ago Earth rotated 2 times faster!

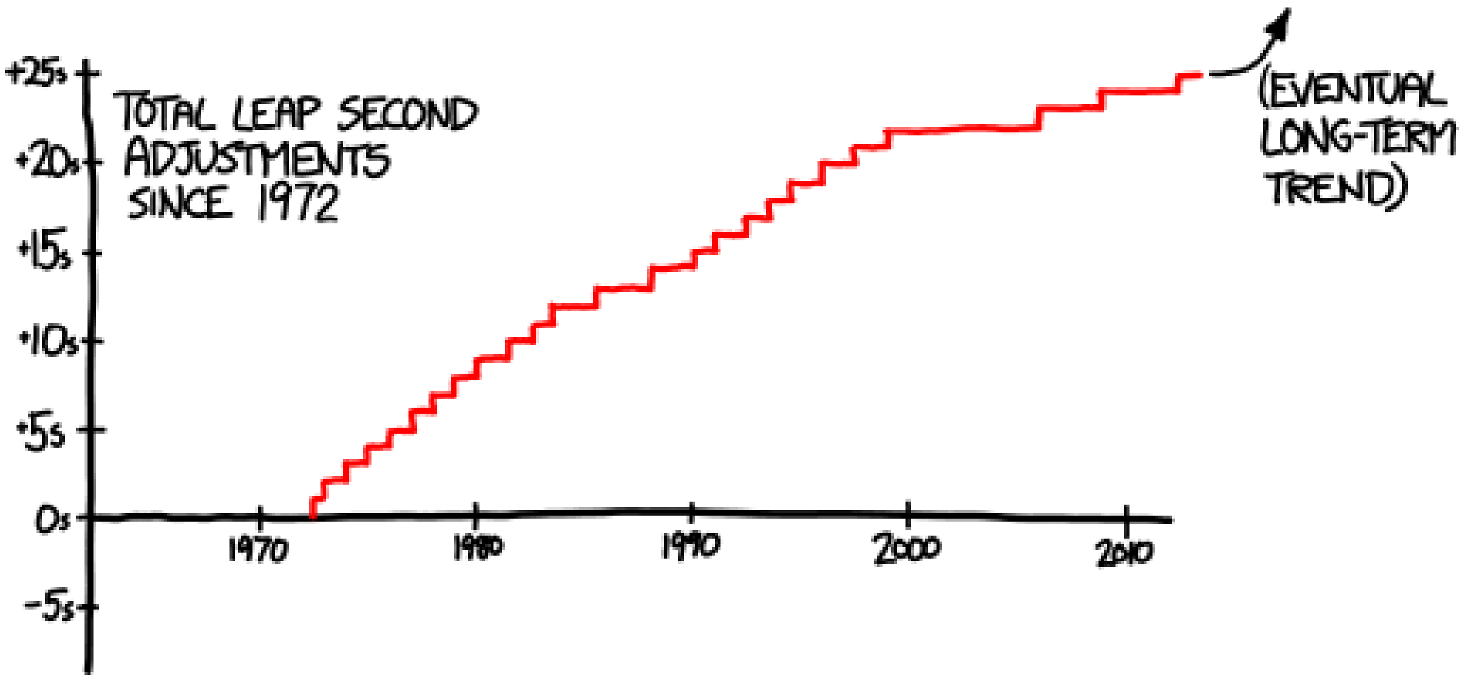






A leap second

Leap second is necessary to adjust the length of the day in respect to the atomic time scale



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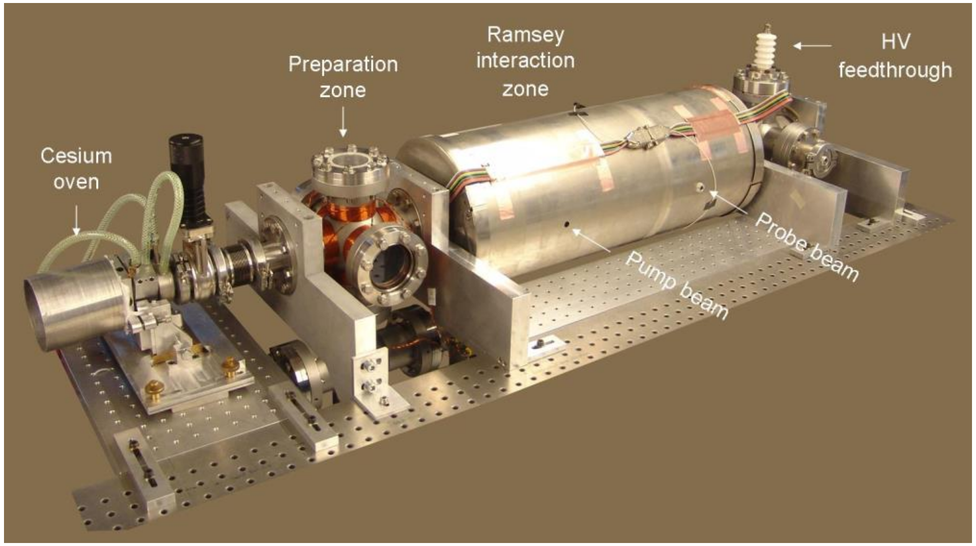
Microwave atomic clock (Cs beam clock and Cs fountain)

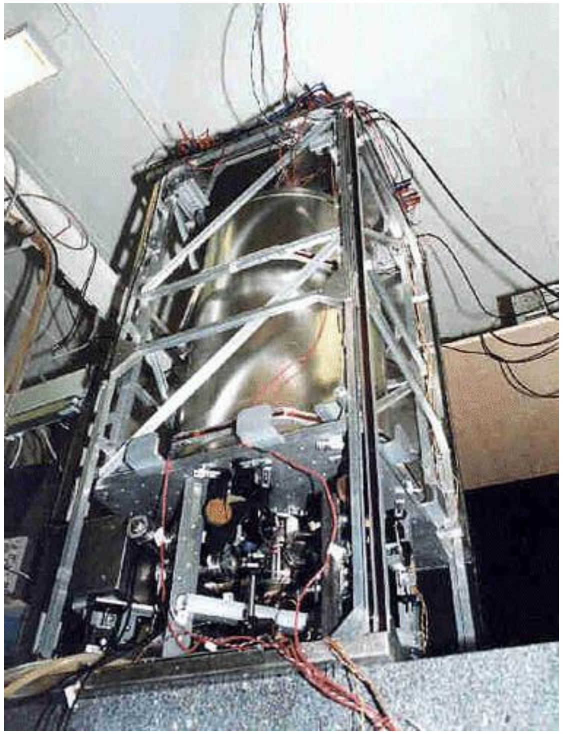


Cs beam clock

$\sim 10^{-14}$

Cs fountain clock

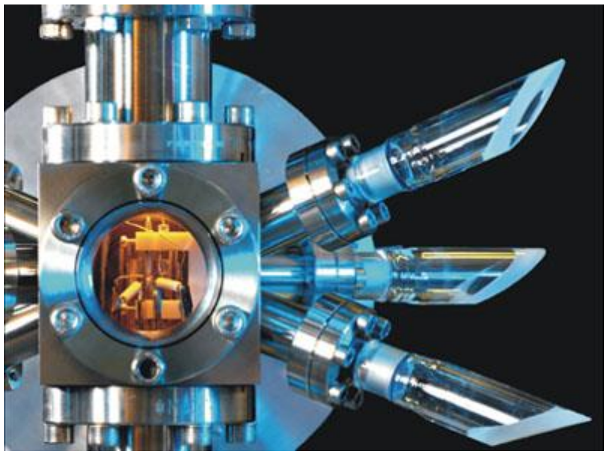
$\sim 10^{-16}$



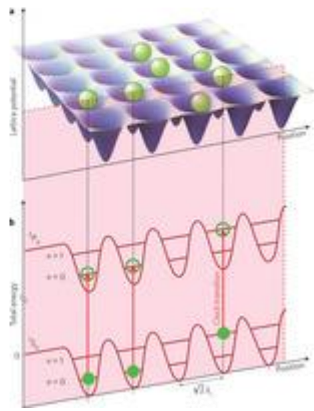
Optical clocks

$$Q = \nu_0/\Delta\nu$$

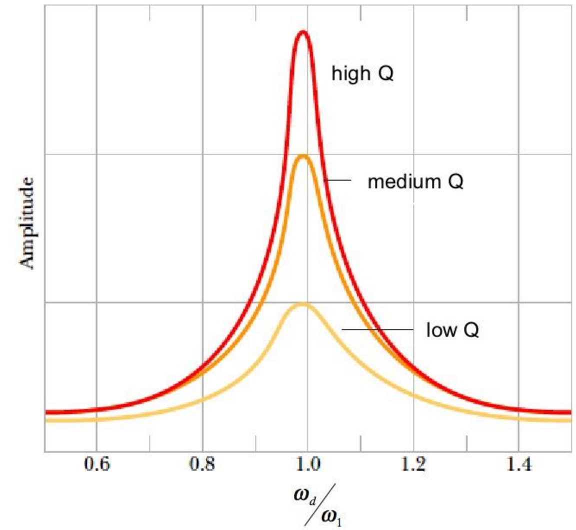
Instability <10^{-17}



Trapped ions



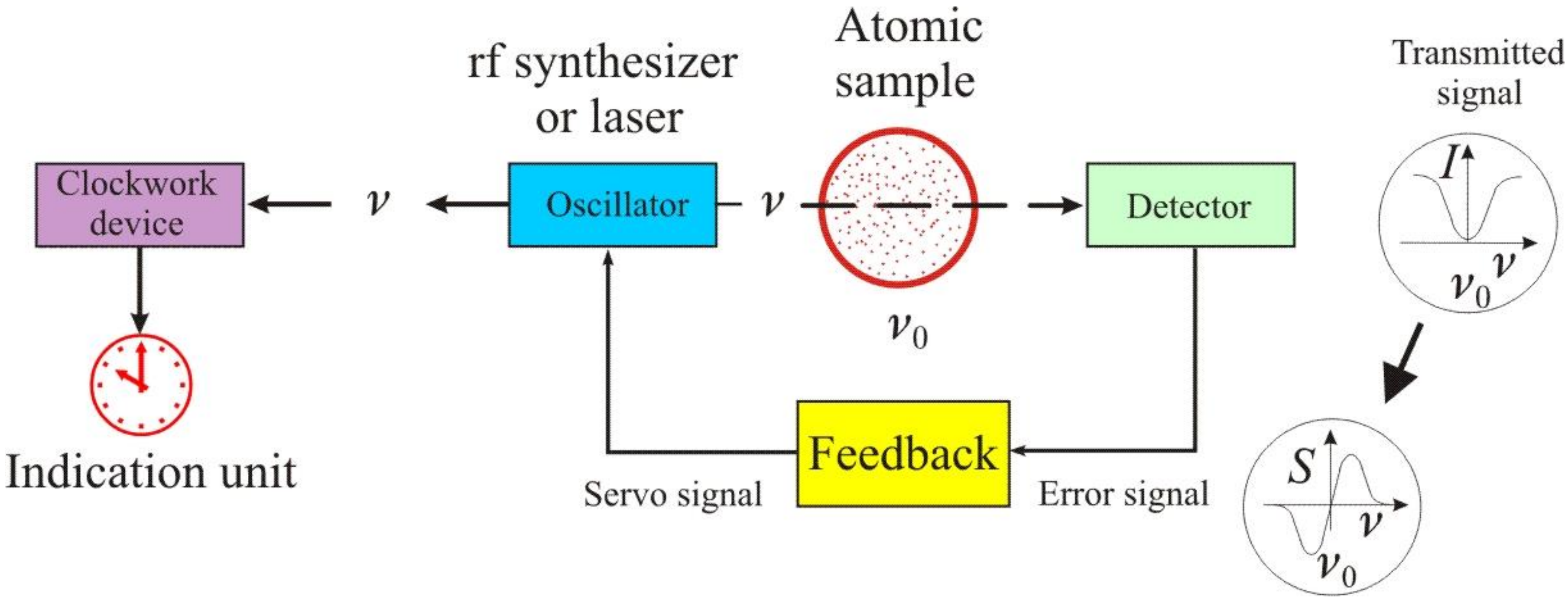
Optical lattice



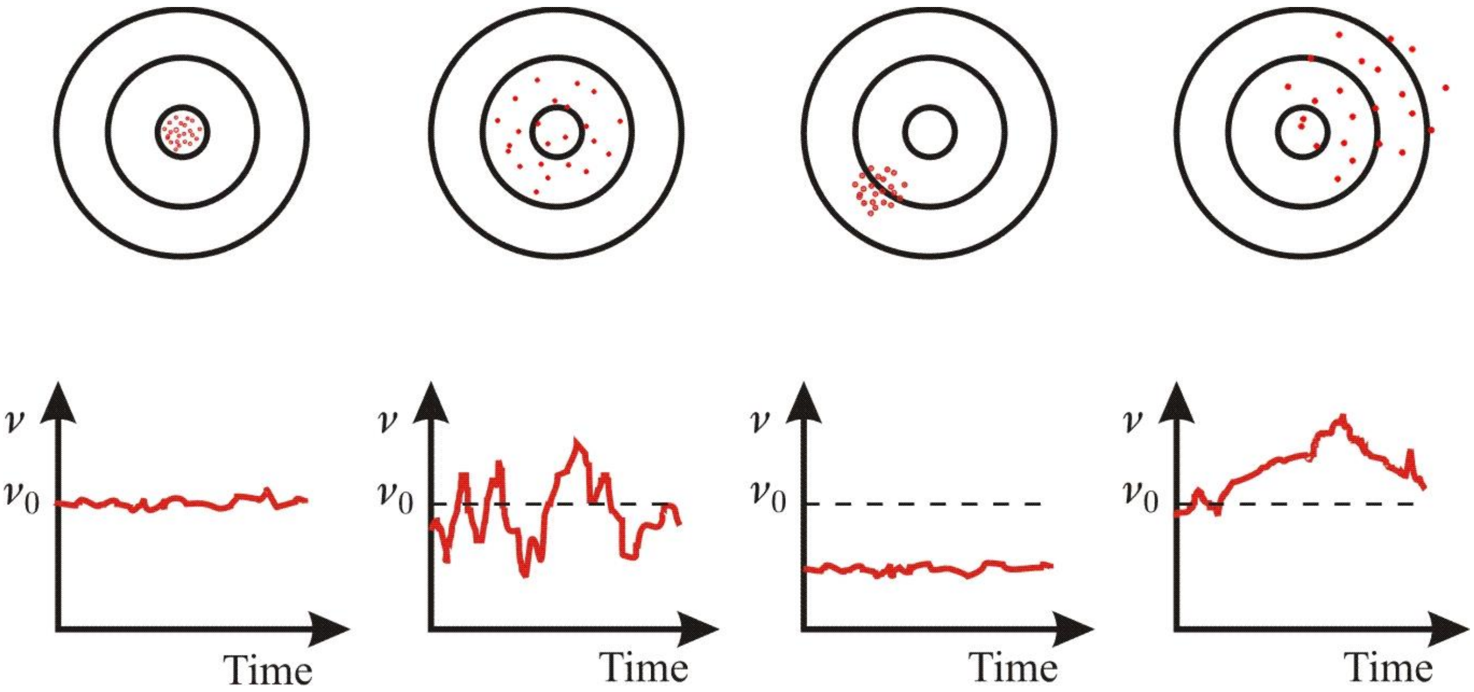
- mechanical: $\nu_0 \sim 1\,\mathrm{Hz}$
- quartz: $\nu_0 \sim 10^7\,\mathrm{Hz}$
- microwave: $\nu_0 \sim 10^{10}\,\mathrm{Hz}$

Further? **Optical!** $\nu_0 \sim 10^{15}\,\mathrm{Hz}$.

Atomic clock schematics



Accuracy and stability



Accurate	+	+	-	-
Stable	+	-	+	-

Oscillator. Modulated amplitude and phase

Harmonic oscillator equation

$$U(t) = U_0 \cos(\omega_0 t + \phi)$$

Harmonic oscillator with varying amplitude and phase

$$U(t) = U_0(t)\,\cos\varphi(t) = [U_0 + \Delta\, U_0(t)]\cos[\omega_0 t + \phi(t)].$$

Relation between phase and frequency

$$\nu(t) \equiv \frac{1}{2\pi}\frac{d\varphi(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}[2\pi\nu_0 t + \phi(t)] = \nu_0 + \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

Damped oscillations

$$U(t) = U_0\, e^{-\frac{\Gamma}{2}t} \cos \omega_0 t$$

Fourier transformation:

$$A(\omega) = \int_0^{\infty} U_0\, e^{-\frac{\Gamma}{2}t}\, \cos(\omega_0 t)\, e^{-i\omega t}\, dt$$

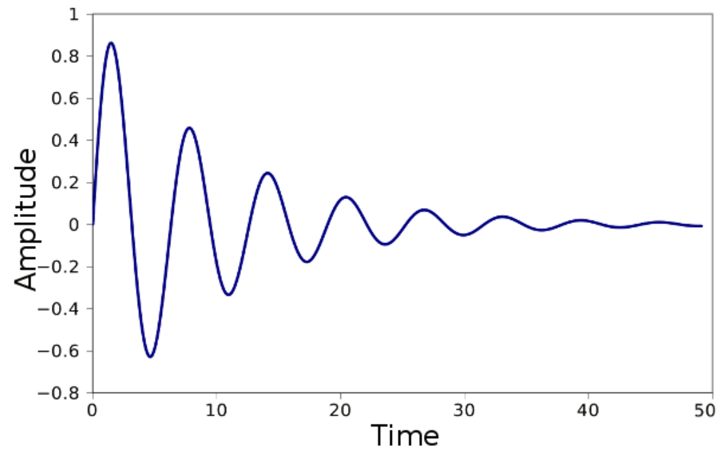
$$A(\omega) = \frac{U_0}{2} \frac{-i(\omega - \omega_0) + \frac{\Gamma}{2}}{(\omega - \omega_0)^2 + (\frac{\Gamma}{2})^2}$$

Power spectrum

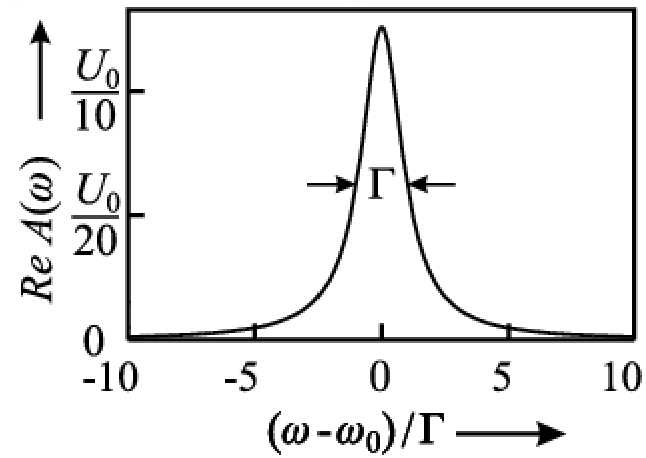
$$P(\omega) = \frac{U_0^2}{4} \frac{1}{(\omega - \omega_0)^2 + (\frac{\Gamma}{2})^2}$$

The Lorentzian function!

$$Q = \frac{\omega_0}{\Gamma} = \frac{\omega_0}{\Delta\omega}$$

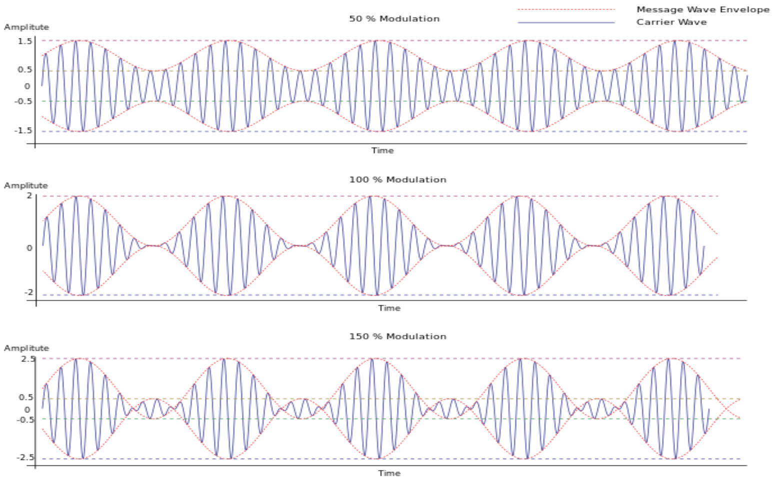


Spectrum of damped oscillations



Harmonic amplitude modulation

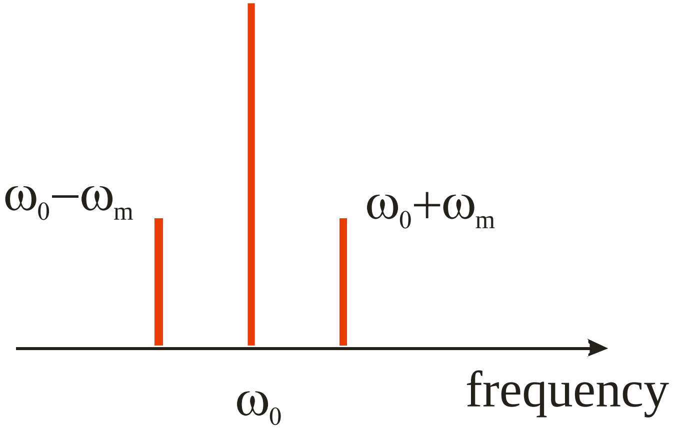
$$U_{AM}(t) = (U_0 + \Delta U_0 \cos \omega_m t) \cos \omega_0 t$$
$$= U_0 (1 + M \cos \omega_m t) \cos \omega_0 t \,,$$



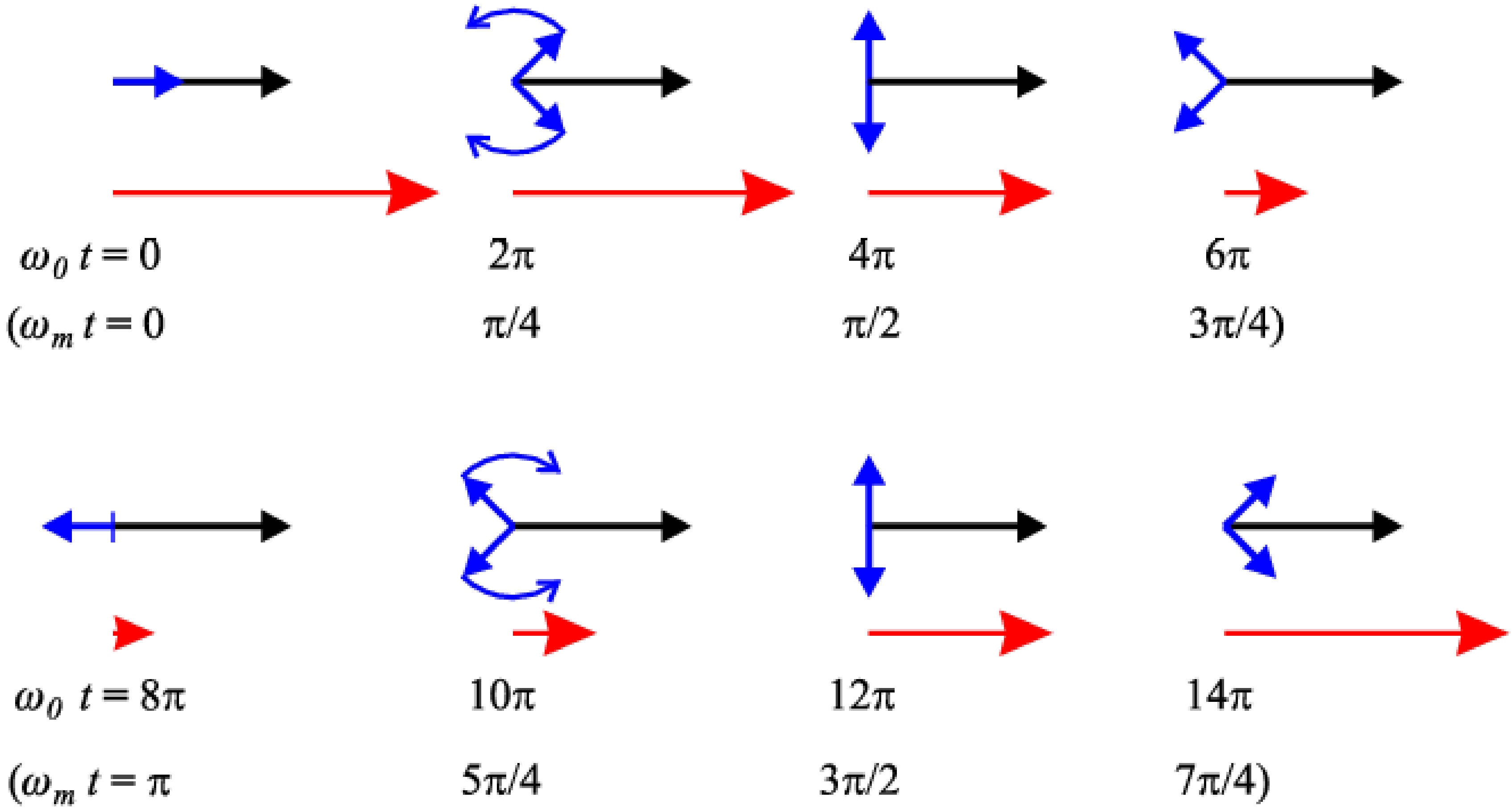
M – modulation index

Signal spectrum

$$U_{AM}(t) = U_0 \left[\cos \omega_0 t + \frac{M}{2} \cos(\omega_0 + \omega_m) t + \frac{M}{2} \cos(\omega_0 - \omega_m) t \right]$$



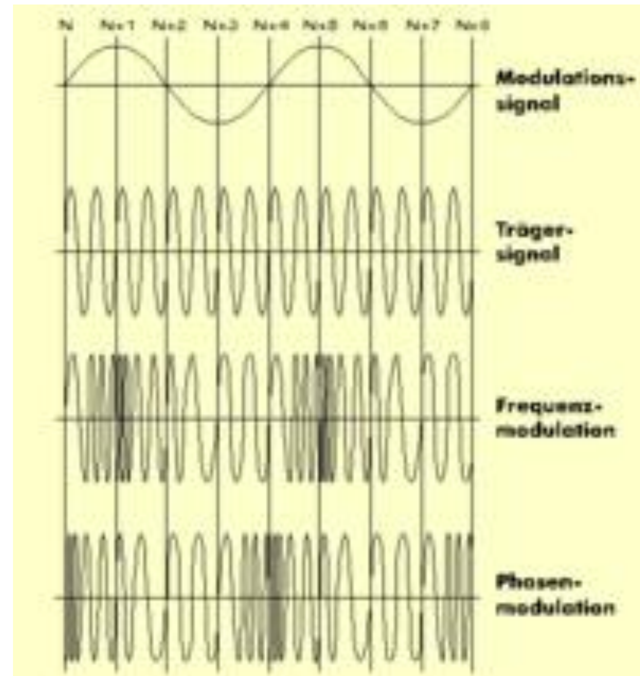
Phase plane representation



Harmonic phase/frequency modulation

$$U_{\rm PM}(t) = U_0 \cos\varphi = U_0 \cos(\omega_0 t + \delta \cos \omega_m t)$$

δ – phase modulation index

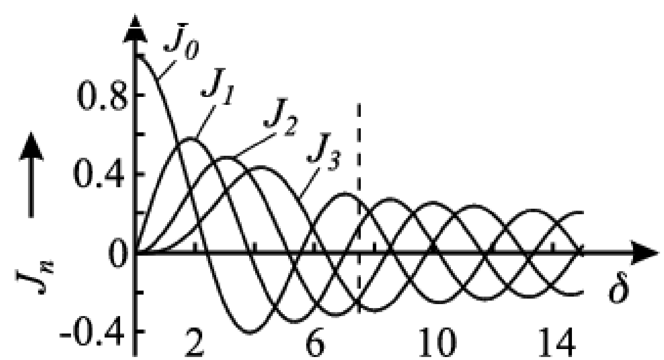


Signal spectrum

$$U_{\rm PM}(t) = U_0 \sum_{n=-\infty}^{\infty} \Re\{(i)^n J_n(\delta)\, \exp[i(\omega_0 + n\,\omega_m)\,t]\}$$

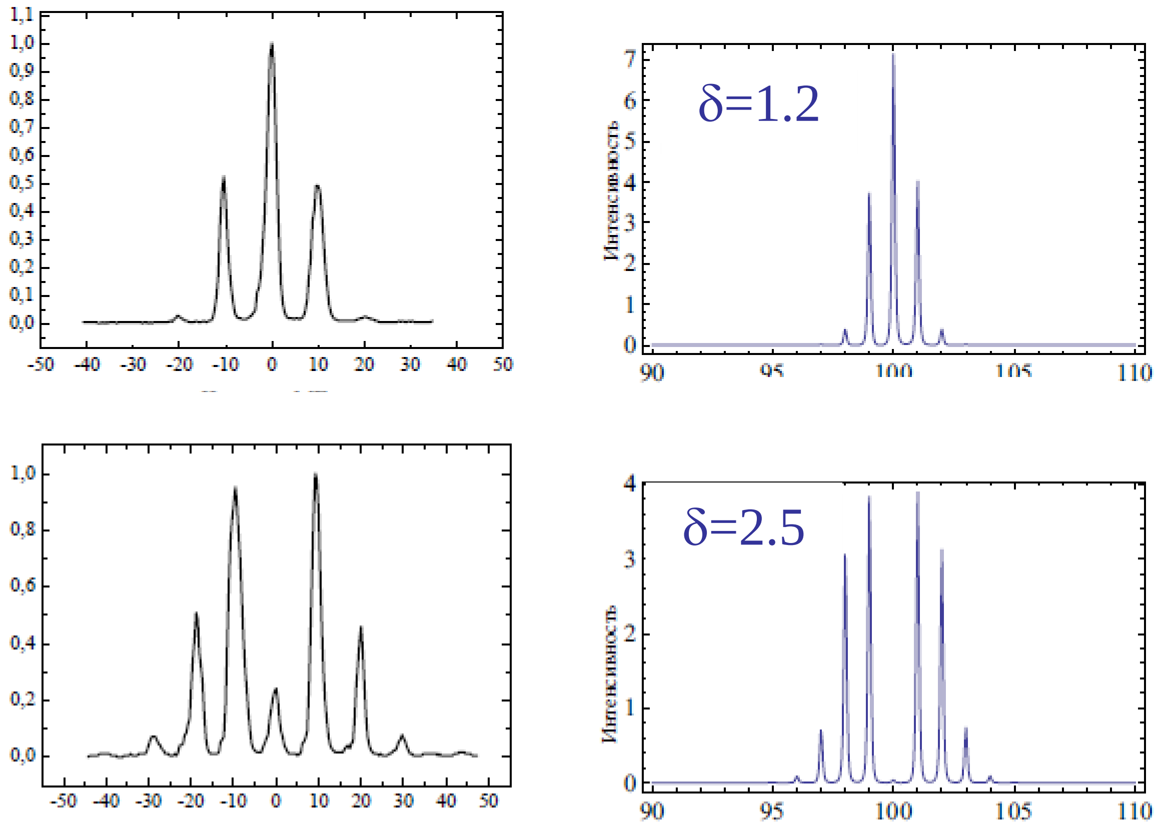
J_n are the Bessel functions:

$$\begin{aligned}
J_0(\delta) &= 1 - \left(\frac{\delta}{2}\right)^2 + \frac{1}{4}\left(\frac{\delta}{2}\right)^4 - \frac{1}{36}\left(\frac{\delta}{2}\right)^6 + \cdots \\
J_1(\delta) &= \left(\frac{\delta}{2}\right) - \frac{1}{2}\left(\frac{\delta}{2}\right)^3 + \frac{1}{12}\left(\frac{\delta}{2}\right)^5 - \cdots \\
J_2(\delta) &= \frac{1}{2}\left(\frac{\delta}{2}\right)^2 - \frac{1}{6}\left(\frac{\delta}{2}\right)^4 + \frac{1}{48}\left(\frac{\delta}{2}\right)^6 - \cdots \\
J_3(\delta) &= \frac{1}{6}\left(\frac{\delta}{2}\right)^3 + \frac{1}{24}\left(\frac{\delta}{2}\right)^5 + \frac{1}{240}\left(\frac{\delta}{2}\right)^7 - \cdots
\end{aligned}$$



Spectra of phase modulated signal

Signal



Frequency Frequency

Phase plane representation