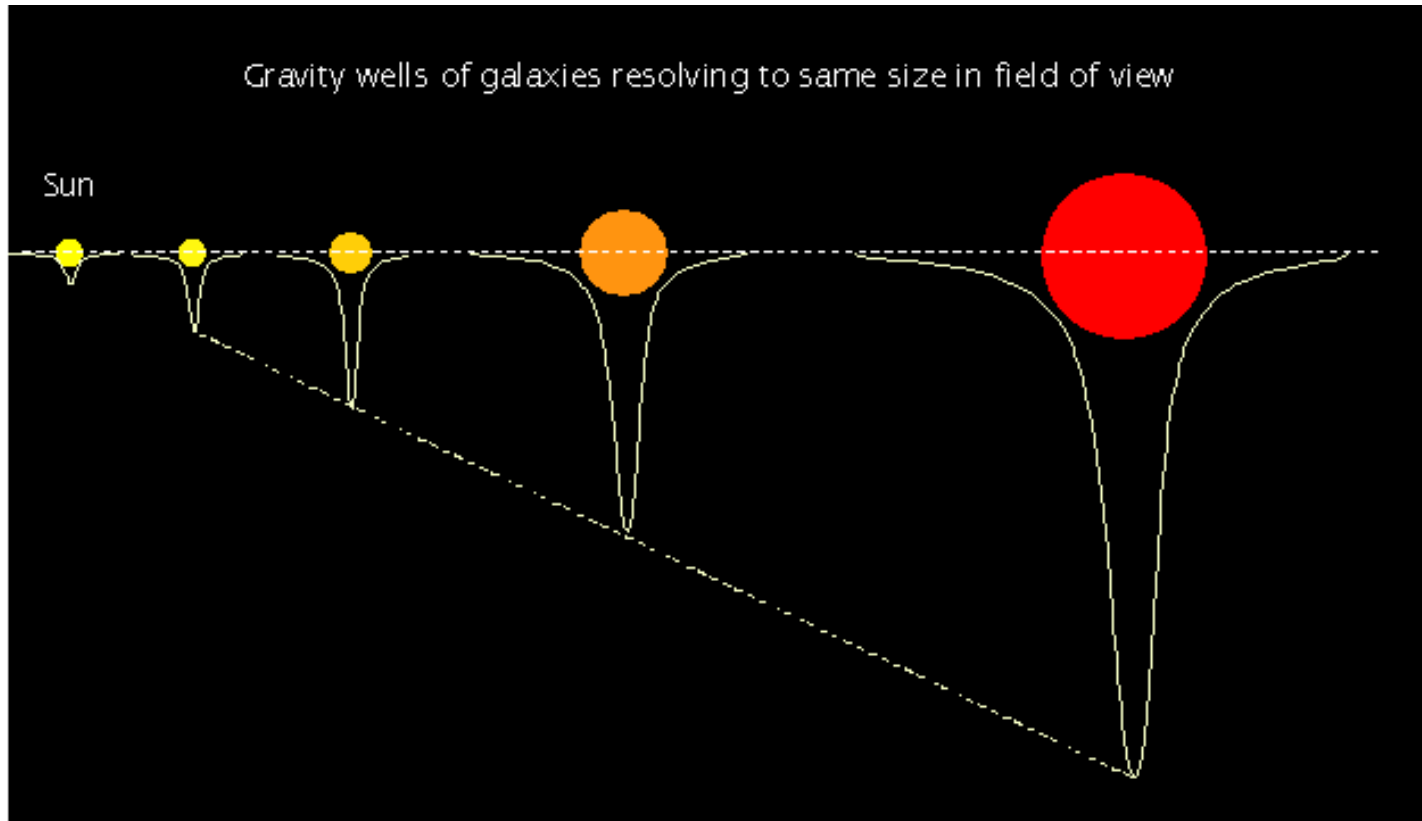


## Lecture 4

- Space and time in Einstein's theory of gravitation, basics of General relativity
- Time transformation in rotating frame, gravitational red shift, time dilation, Sagnac effect.
- Methods of time and frequency transfer, clock synchronization.



# Space-time curvature

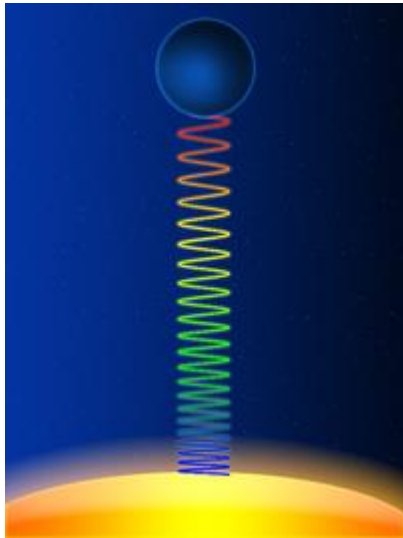


# Fractional frequency shift

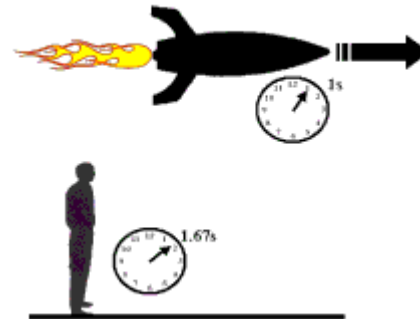
Non-rotating frame

$$h(t) = \frac{U(t)}{c^2} + \frac{v^2}{2c^2} + \mathcal{O}\left(\frac{1}{c^4}\right)$$

Gravitational red shift

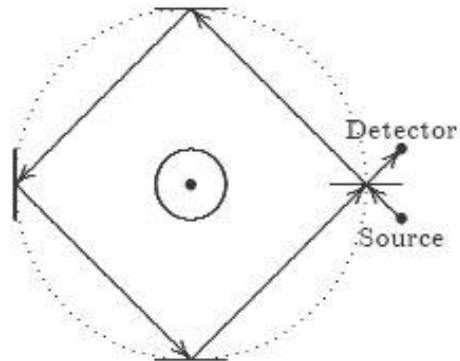


Time dilation

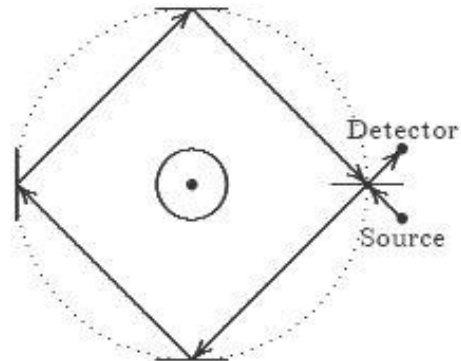


# The Sagnac effect

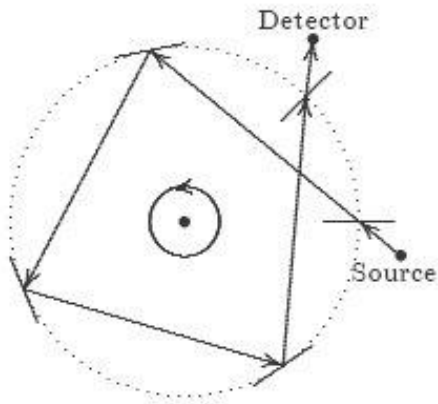
a) Non-rotating loop  
positive path



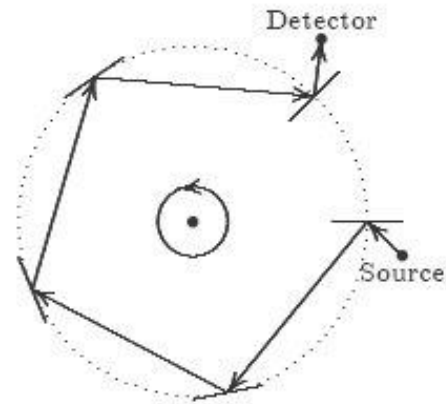
b) Non-rotating loop  
negative path



c) Rotating loop  
positive path



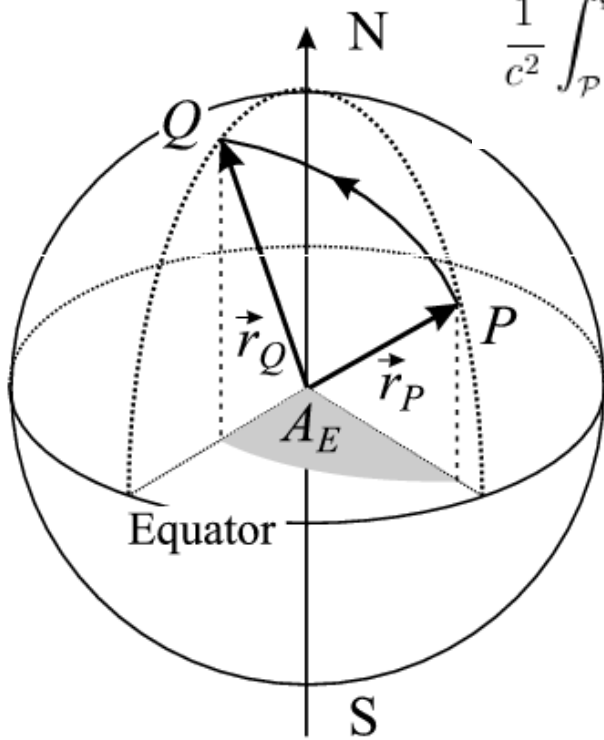
d) Rotating loop  
negative path



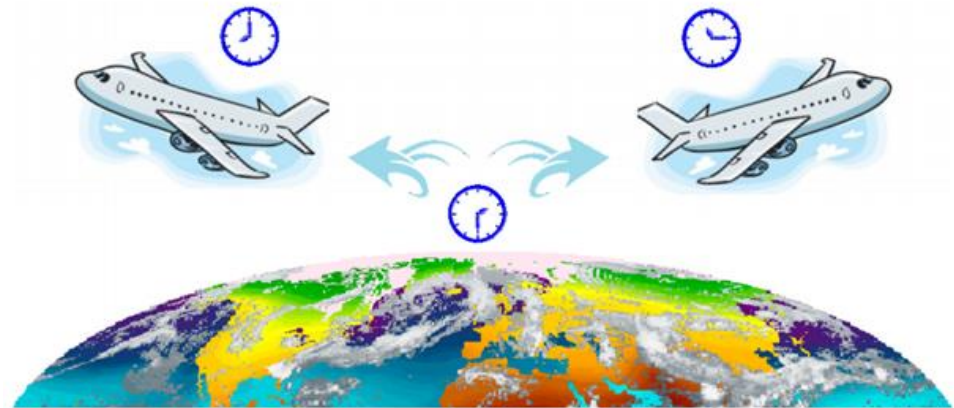
## Rotating frame: the Sagnac effect

$$h(t) = \frac{1}{c^2} \left[ U_g + \Delta U(t) + \frac{V(t)^2}{2} \right] + \frac{2\omega}{c^2} \frac{dA_E}{dt}$$

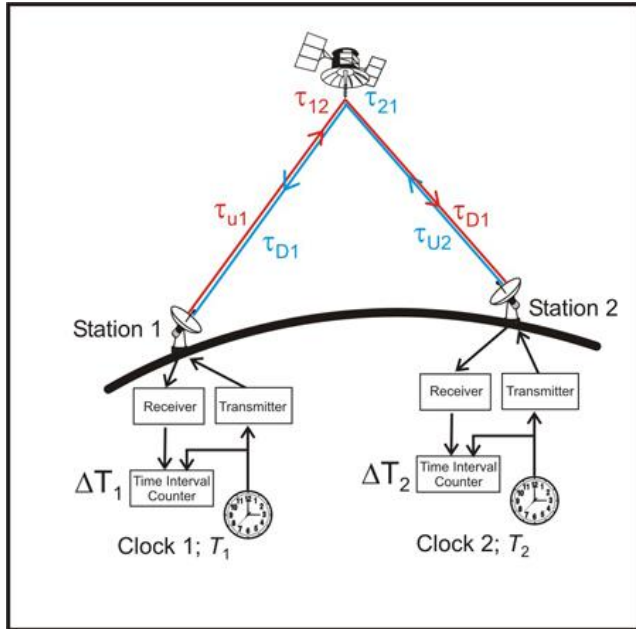
$$\frac{1}{c^2} \int_P^Q (\vec{\omega} \times \vec{r}') \cdot d\vec{r}' = \frac{1}{c^2} \int_P^Q \vec{\omega} \cdot (\vec{r}' \times d\vec{r}') = 2 \frac{1}{c^2} \int_P^Q \vec{\omega} \cdot d\vec{A}_E = \frac{2\omega A_E}{c^2}$$



Hafele and Keating Experiment

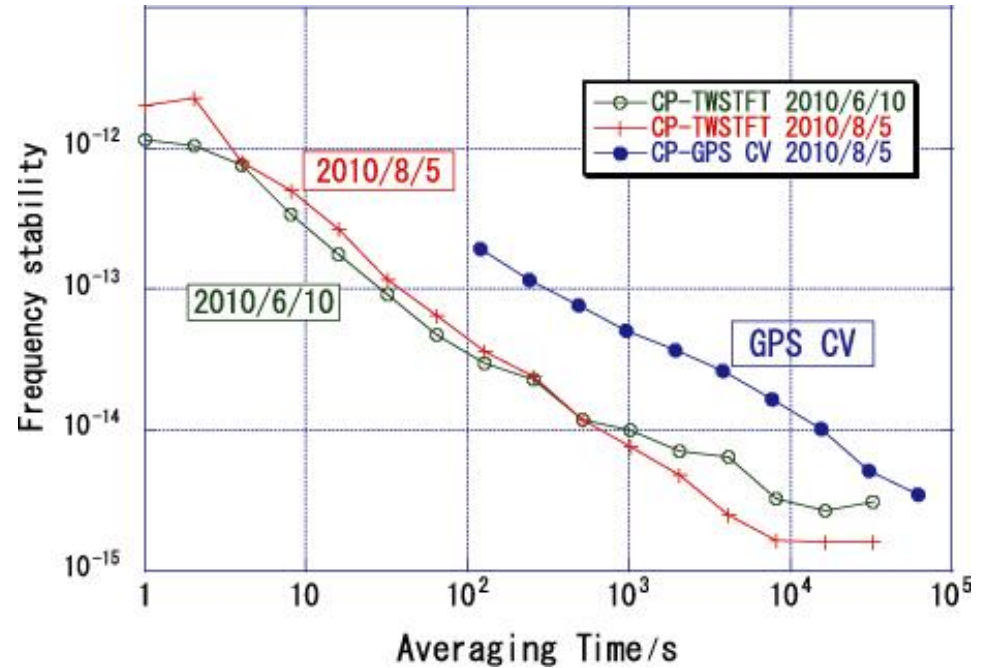


# Two-way time and frequency transfer



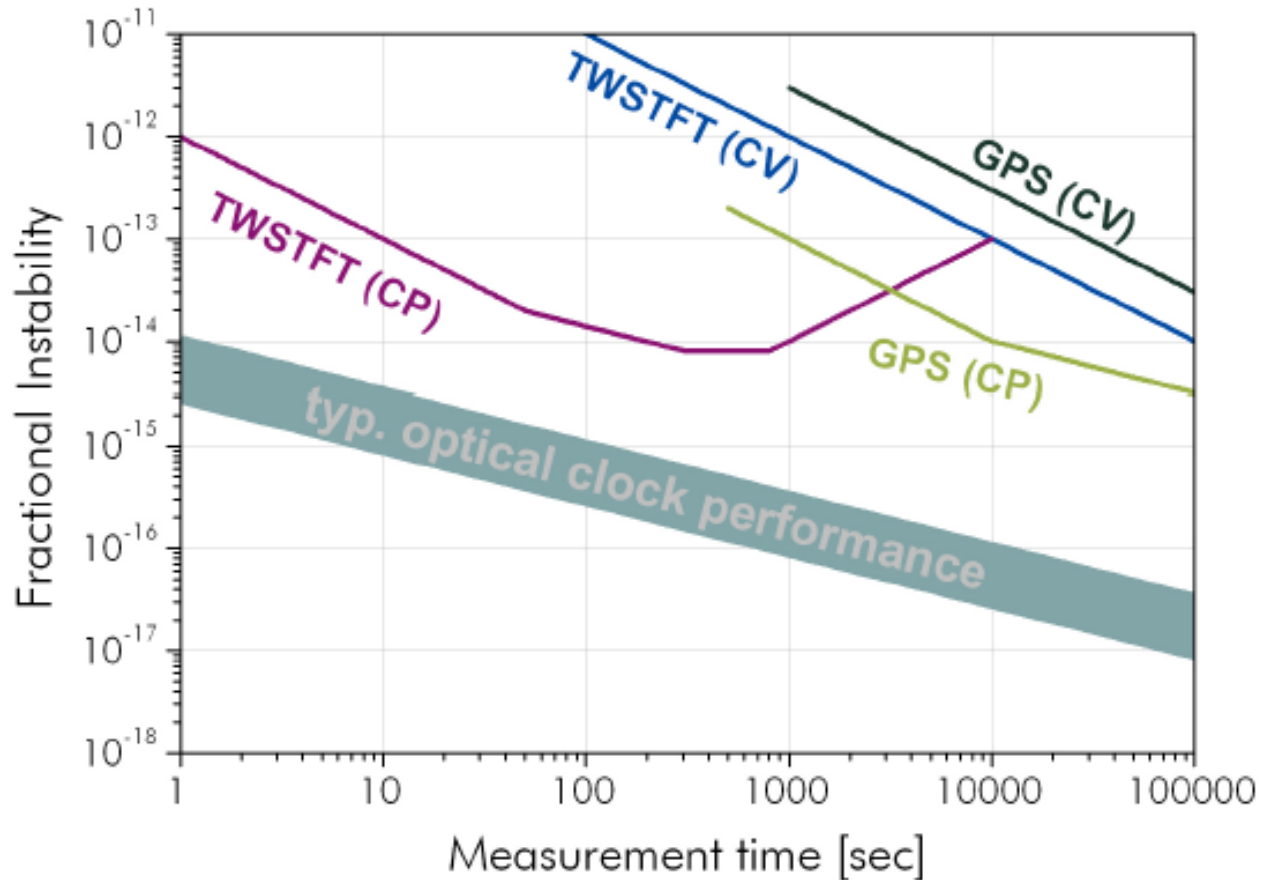
$$\Delta t_A = t_A - t_B + \delta_{B \rightarrow A}$$

$$\Delta t_B = t_B - t_A + \delta_{A \rightarrow B}$$

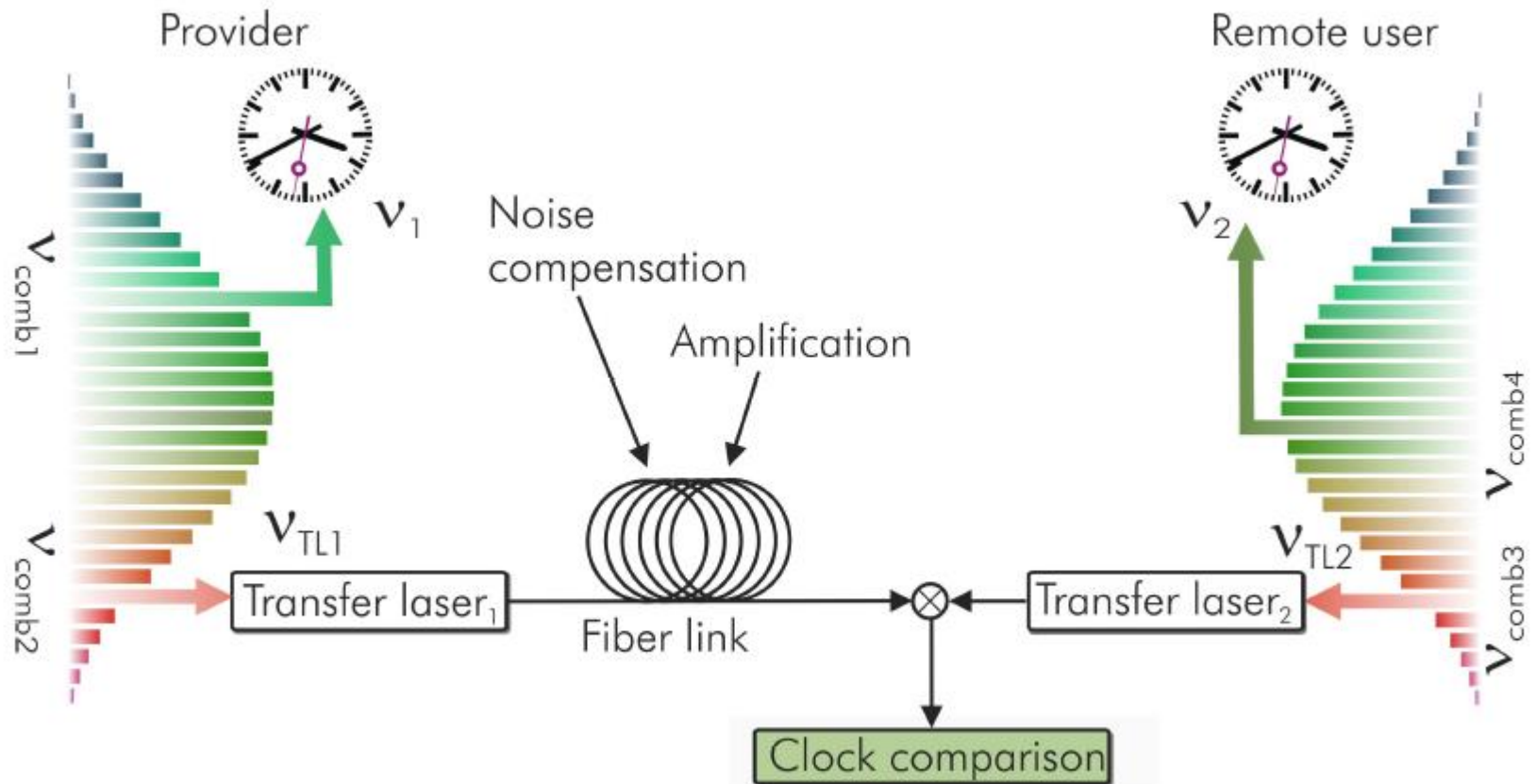




# Today's demands for frequency transfer

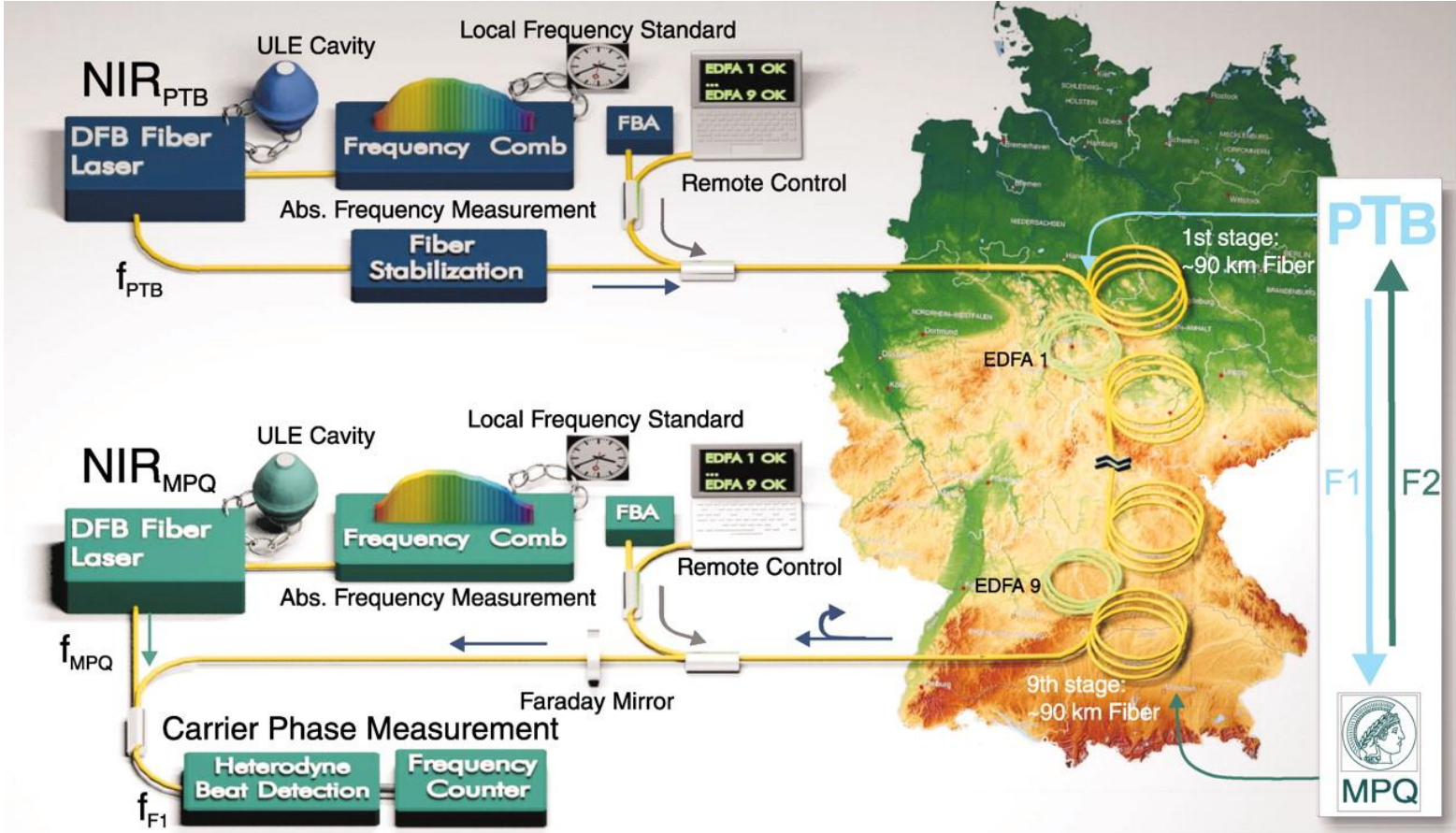


# Optical frequency transfer



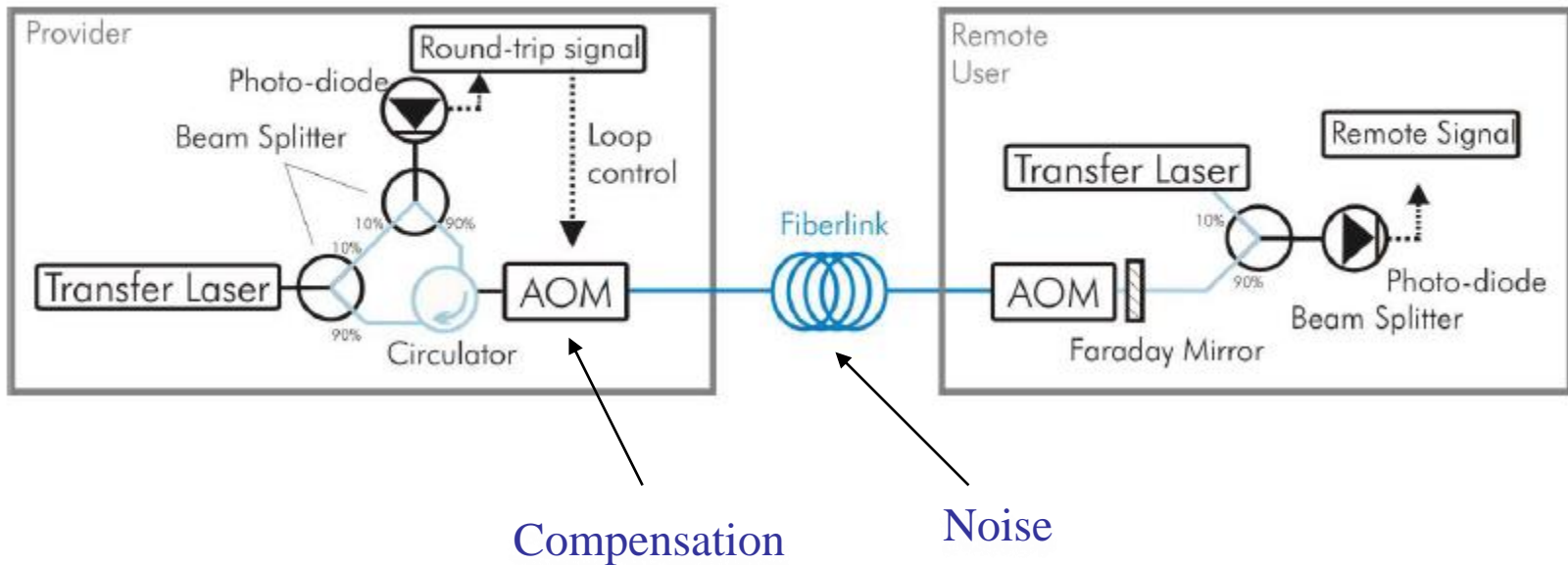


# The 920-km fiber link

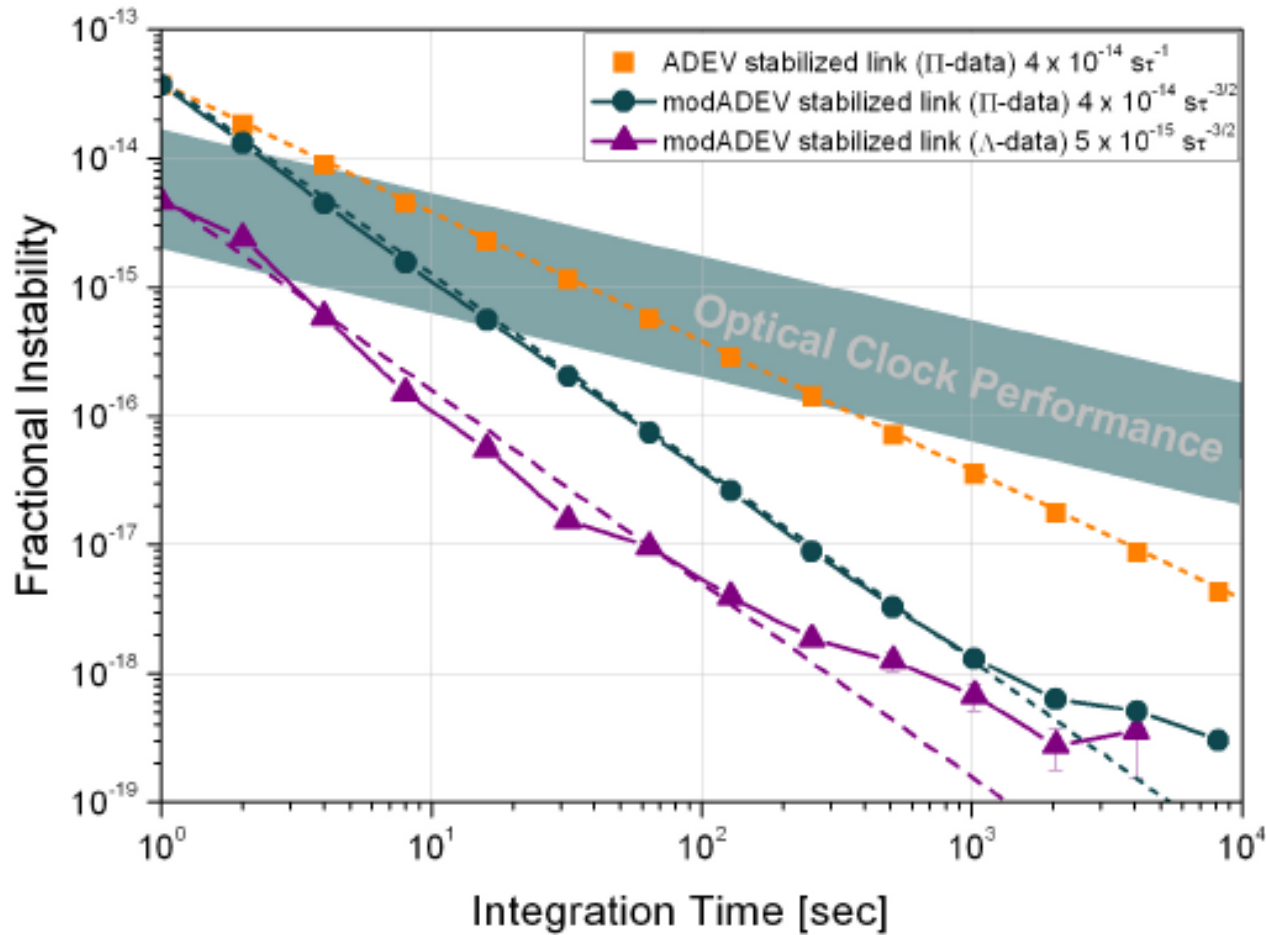


# Stabilization loop

## Mach-Zender interferometer setup



# Stability of the 920-km link



# Future prospects: All European optical fiber link network

