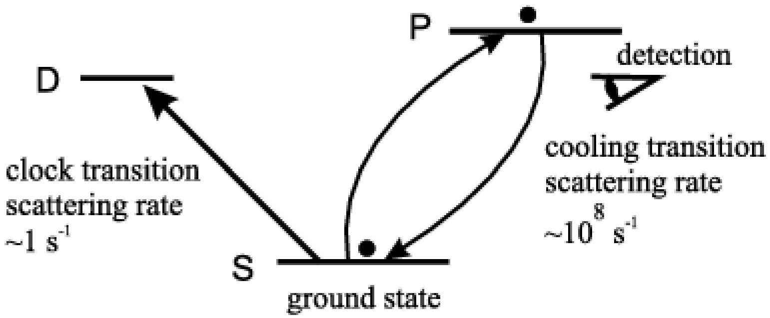
Nikolai Kolachevsky

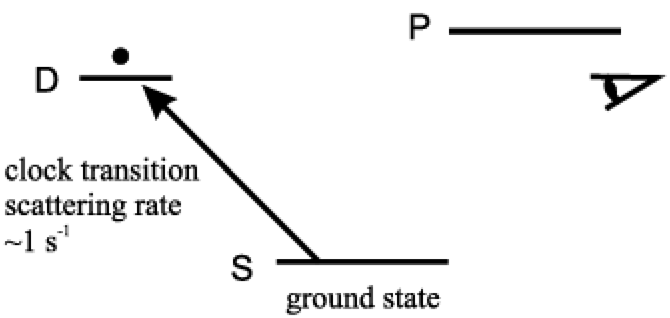
Lecture 12

- Precision measurements in the traps, electron shelving.
- Elements of quantum logic in ion traps. CNOT gate.
- Motional degrees of freedom. Cirac-Zoller gate.
- Information transfer between clock and cooling ions. Precision spectroscopy using quantum logic.

Electron shelving

How to detect transition in a single particle with a very small scattering rate?

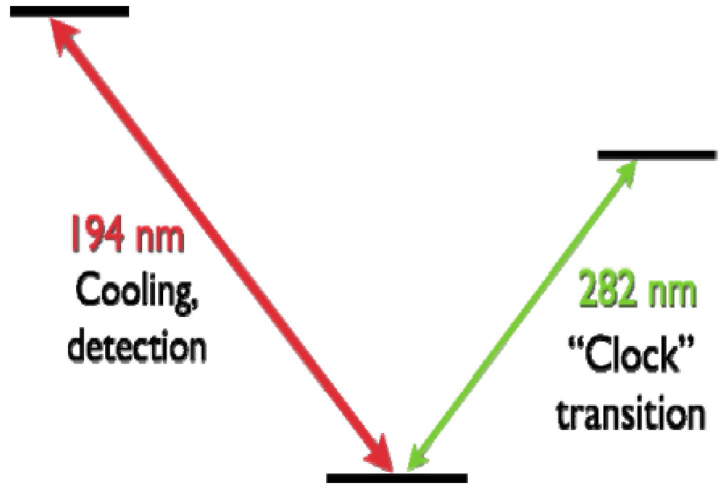


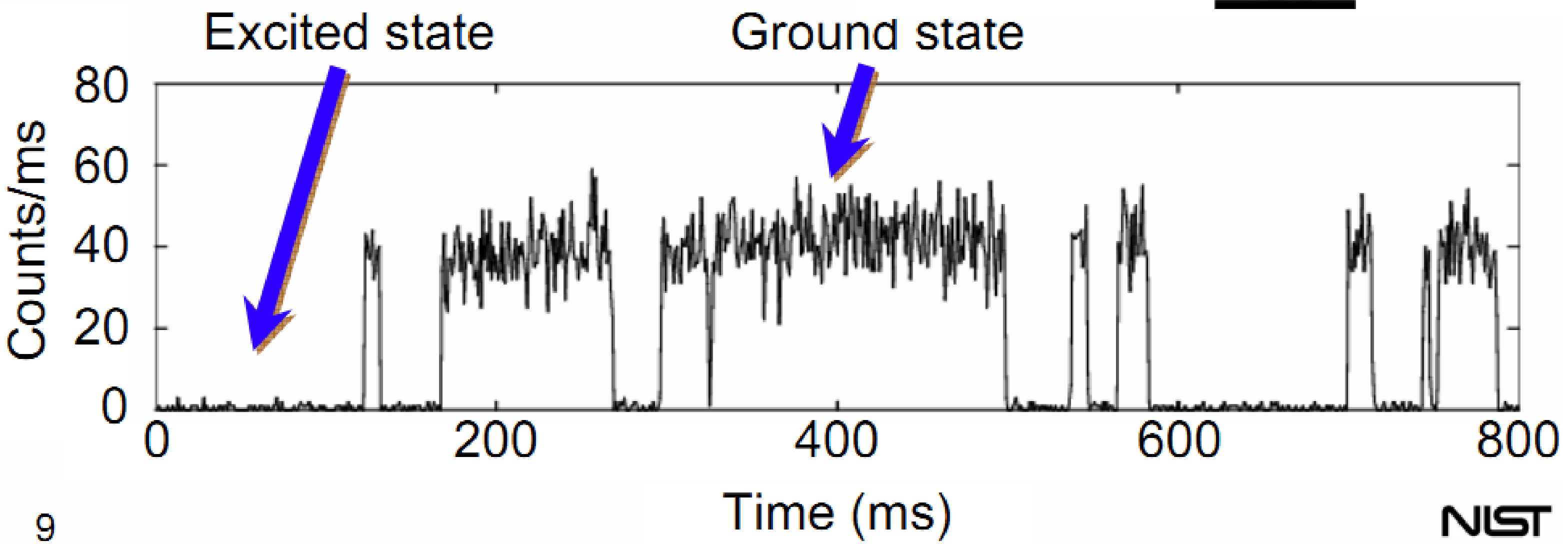


Quantum jumps

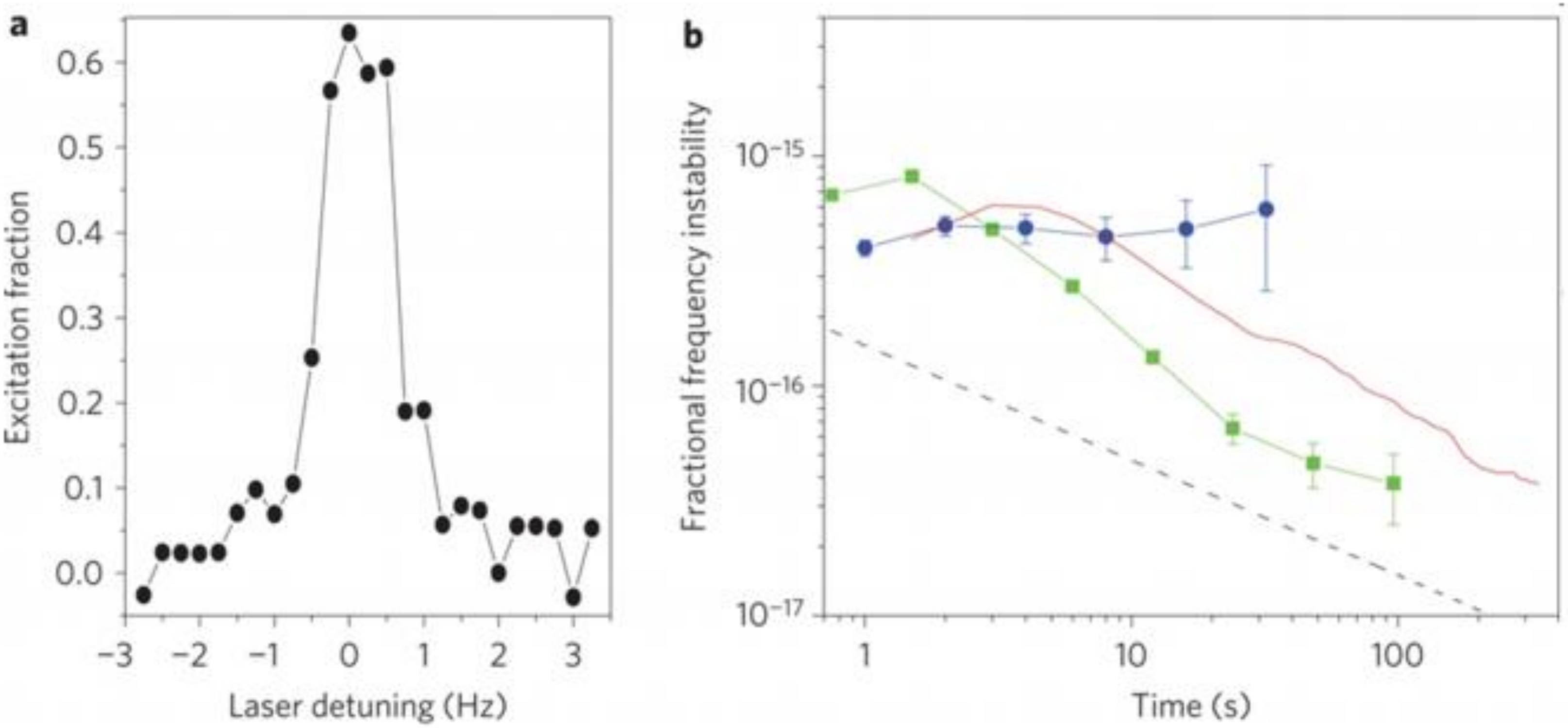
The mercury ion acts as
a noiseless, optical amplifier

One absorption event can prevent
millions of scattering events

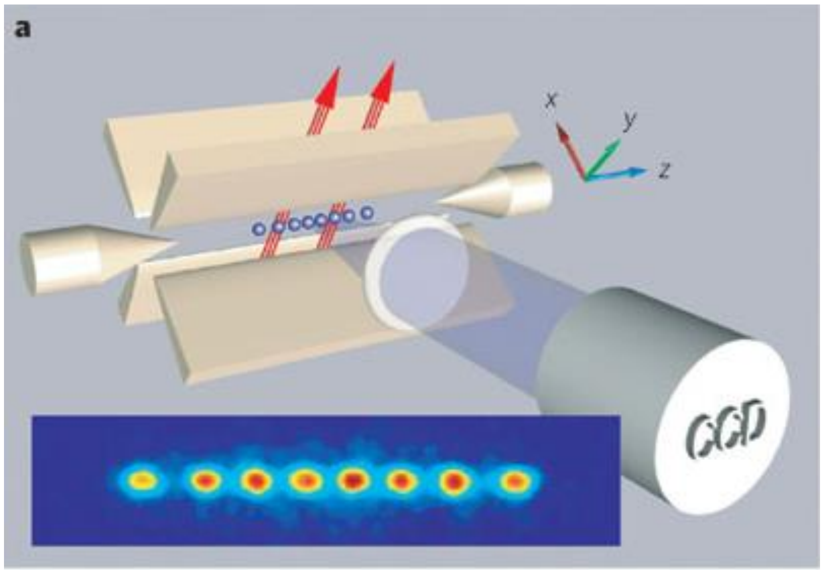


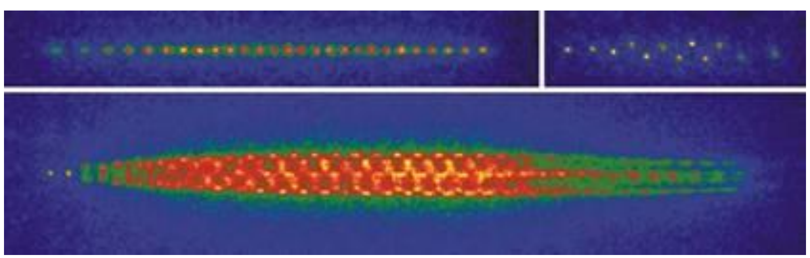


Spectrum of a narrow transition in a single ion



Elements of quantum logic in ion traps





Selectively address

Selectively read out

Scale the number

A Q-bit

$$|\psi\rangle = \alpha|1\rangle + \beta|2\rangle$$

A gate

Example: a π-pulse $\alpha|1\rangle + \beta|2\rangle \rightarrow \beta|1\rangle + \alpha|2\rangle$

Vector and matrix representations

Q-bit $\begin{vmatrix} \alpha \\ \beta \end{vmatrix}$ Gate: unitary 2x2 matrix

Example: SWAP gate
(π - pulse)

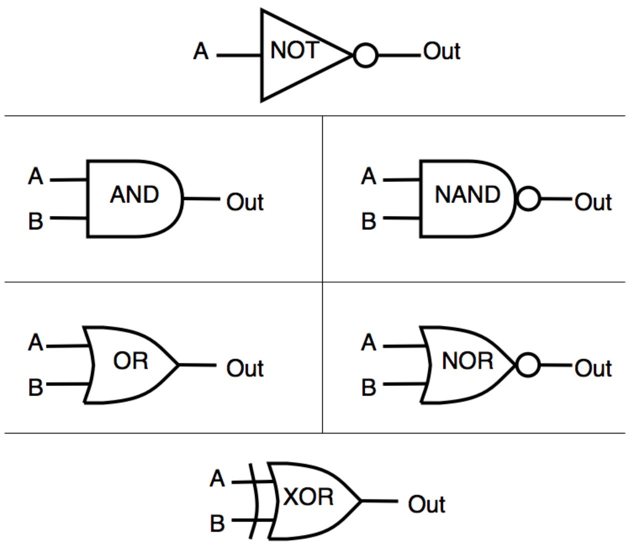
$$X = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \qquad X\begin{vmatrix} \alpha \\ \beta \end{vmatrix} = \begin{vmatrix} \beta \\ \alpha \end{vmatrix}$$

Hadamard gate
(π/2 - pulse)

$$H = \frac{1}{\sqrt{2}}\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

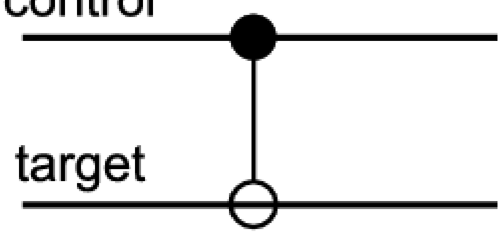
Two-Q-bit gates

Cornerstone of computation



Classical binary gates

Controlled NOT gate (CNOT)



Two Q-bit gate

Control

Target

$$
\begin{aligned}
|00\rangle &\to |00\rangle \\
|01\rangle &\to |01\rangle \\
|10\rangle &\to |11\rangle \\
|11\rangle &\to |10\rangle ,
\end{aligned}
$$

Matrix representation

Two Q-bit CNOT gate

$$U_{\mathrm{CNOT}} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

Two Q-bit PHASE gate

$$U_{\mathrm{phase}} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}$$

Transformation PHASE $\rightarrow$ CNOT

$$U_{\mathrm{CNOT}} = \frac{1}{\sqrt{2}} \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{vmatrix} \cdot U_{\mathrm{phase}} \cdot \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{vmatrix} \frac{1}{\sqrt{2}}$$

Hadamard

Hadamard

Cirac-Zoller gate

Quantum gate proposals

74, NUMBER 20 4091 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*
Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

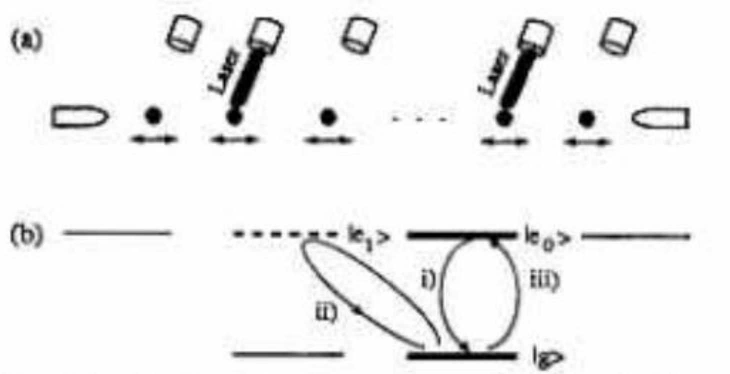


FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

$$ControlNOT: |\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

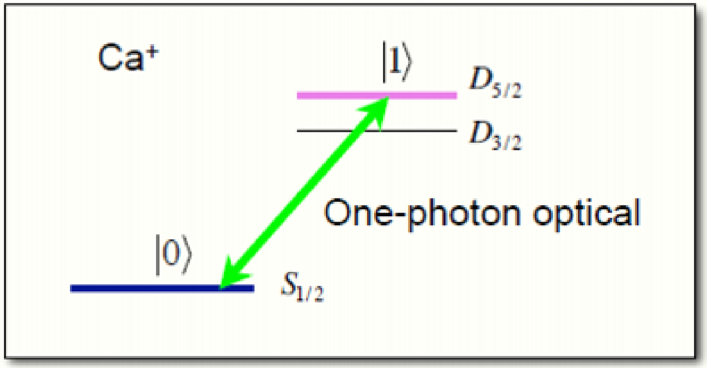
$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

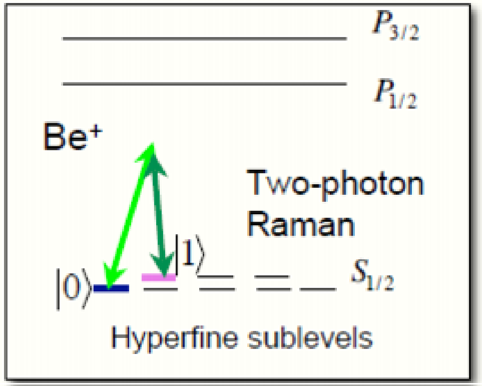
control bit

target bit

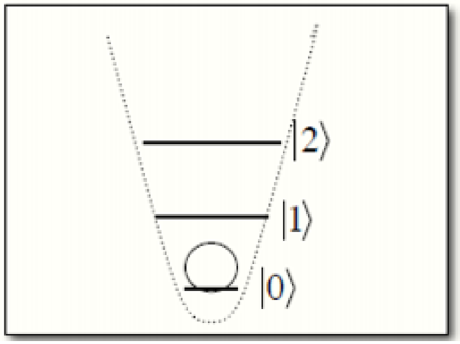
Ion spectrum in the trap

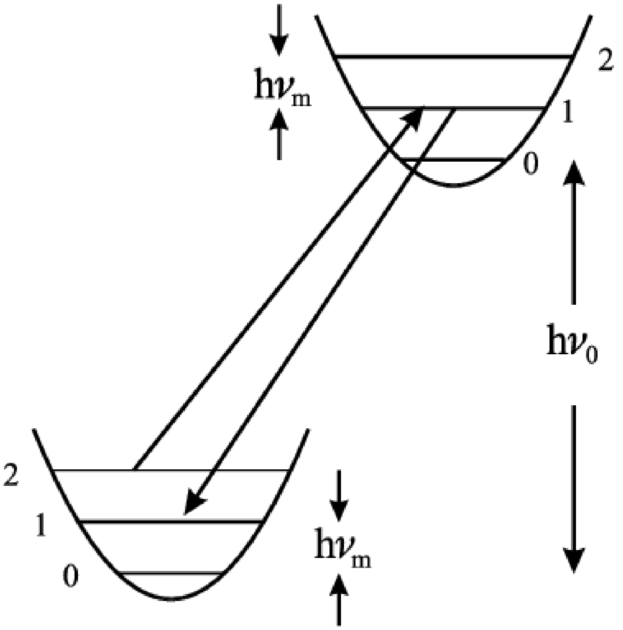
Internal states





Motional states

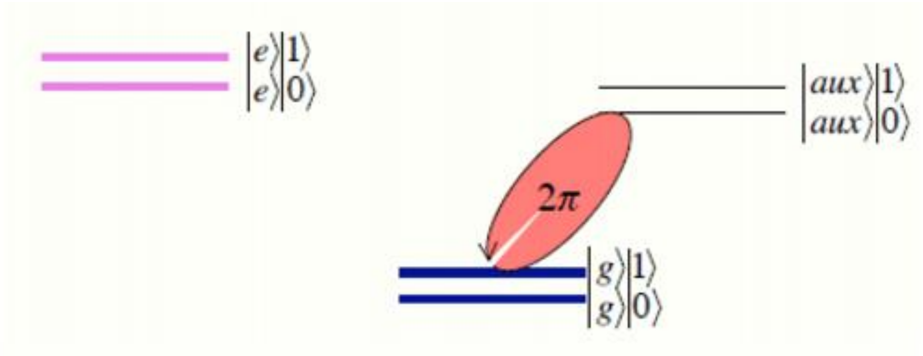




2π pulse on the spin-1/2 system - I

Rotation operator

$$\mathcal{D}(\phi) = \exp(-\frac{i}{\hbar} S_z \phi)$$



If we apply this operator with S_z (spin-1/2 system) to the state $|\alpha\rangle = |+\rangle\langle+|\alpha\rangle + |-\rangle\langle-|\alpha\rangle$ we will get

$$\exp(-\frac{i}{\hbar} S_z \phi)|\alpha\rangle = \exp\left(-\frac{i\phi}{2}\right)|+\rangle\langle+|\alpha\rangle + \exp\left(-\frac{i\phi}{2}\right)|-\rangle\langle-|\alpha\rangle \qquad (12.6)$$

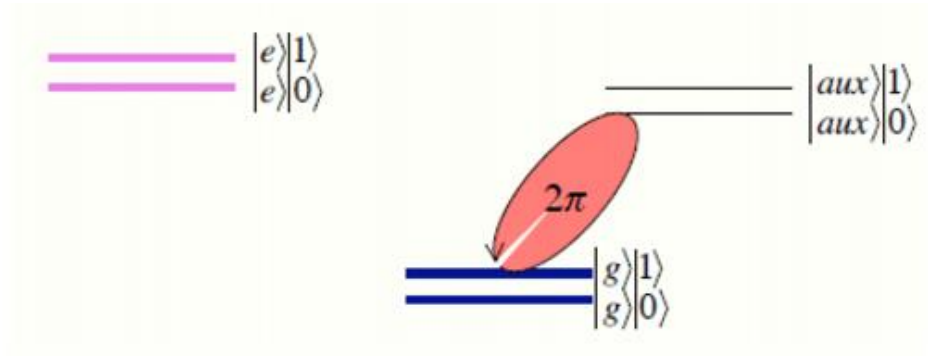
we get the following important relation

$$|\alpha\rangle_{2\pi} = -|\alpha\rangle\,. \qquad (12.7)$$

2π, π pulses on the spin-1/2 system - II

2π-rotation

$$
\begin{aligned}
|g\rangle|0\rangle &\rightarrow |g\rangle|0\rangle \\
|e\rangle|0\rangle &\rightarrow |e\rangle|0\rangle \\
|g\rangle|1\rangle &\rightarrow -|g\rangle|1\rangle \\
|e\rangle|1\rangle &\rightarrow |e\rangle|1\rangle\,.
\end{aligned}
$$



± π-rotation

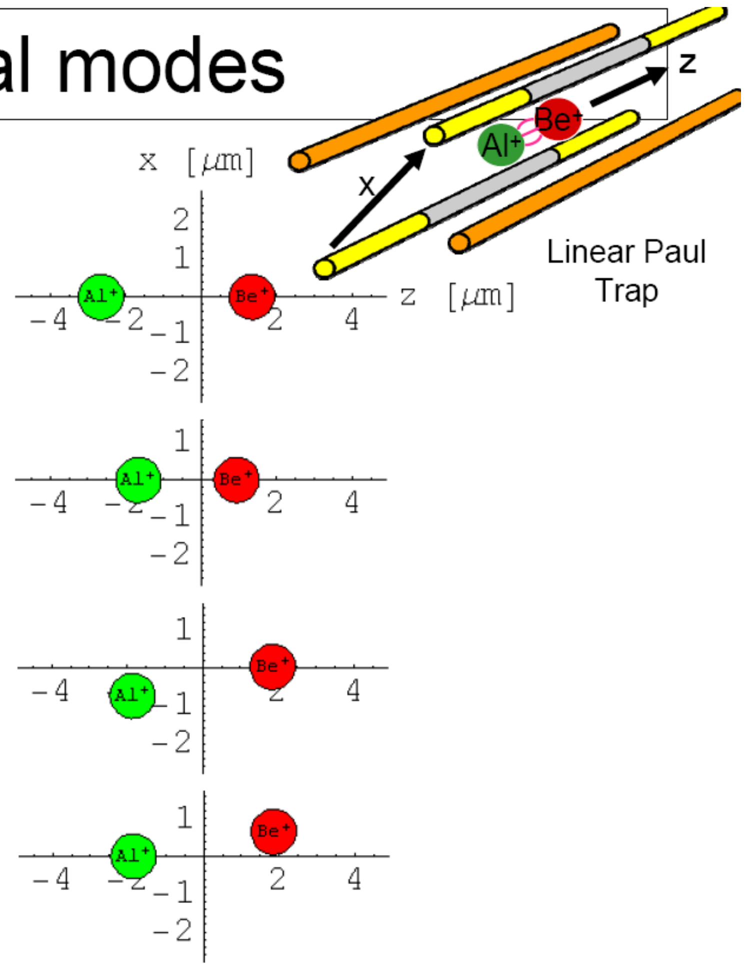
$$
\begin{aligned}
|g\rangle|0\rangle &\rightarrow |g\rangle|0\rangle \\
|e\rangle|0\rangle &\rightarrow |e\rangle|0\rangle \\
|g\rangle|1\rangle &\rightarrow i\,|g\rangle|1\rangle \\
|e\rangle|1\rangle &\rightarrow |e\rangle|1\rangle\,.
\end{aligned}
$$

$$
\begin{aligned}
|g\rangle|0\rangle &\rightarrow |g\rangle|0\rangle \\
|e\rangle|0\rangle &\rightarrow |e\rangle|0\rangle \\
|g\rangle|1\rangle &\rightarrow -i\,|g\rangle|1\rangle \\
|e\rangle|1\rangle &\rightarrow |e\rangle|1\rangle\,.
\end{aligned}
$$

Collective vibrational modes

Normal modes

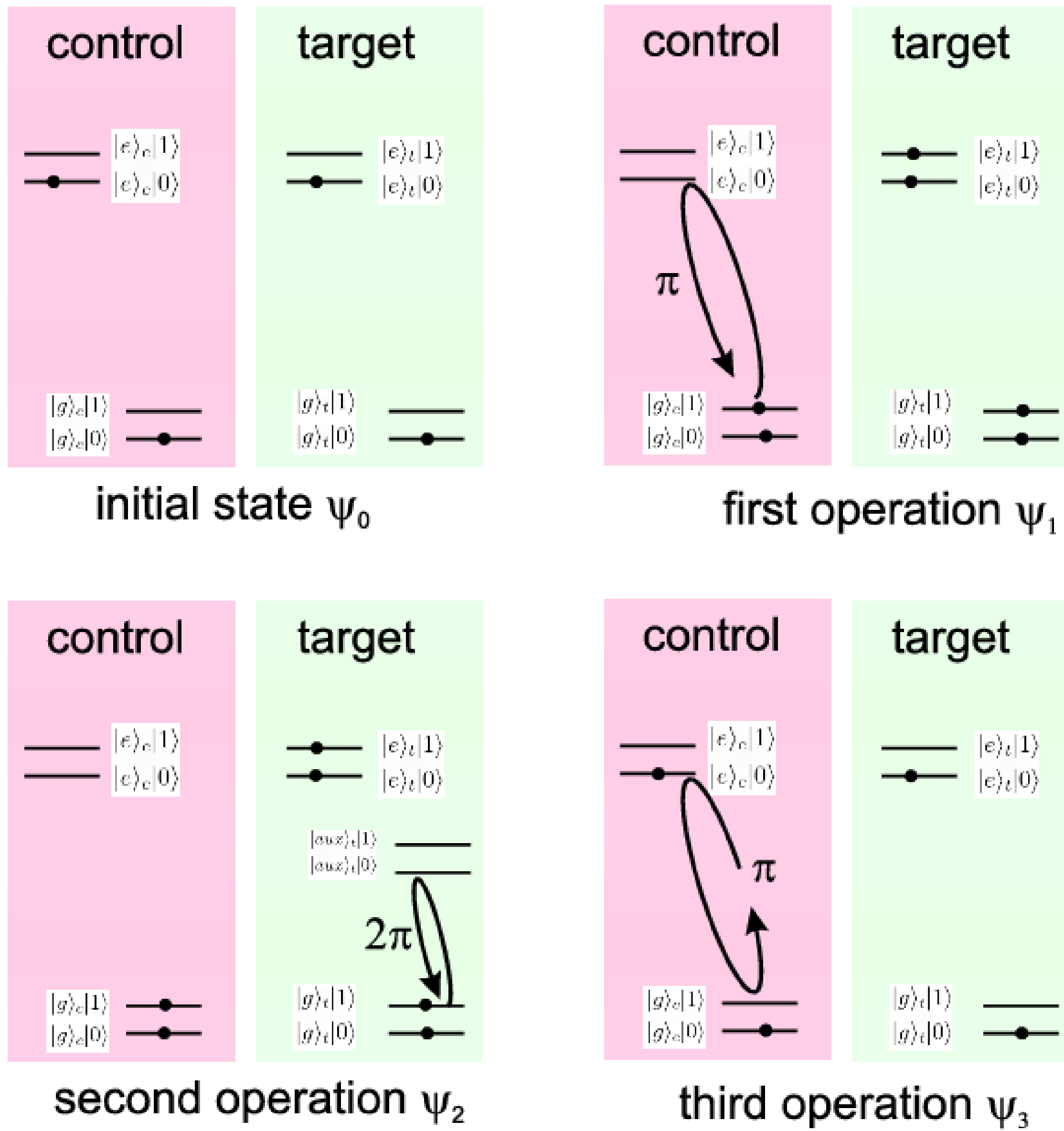
Center-of-mass (COM)	2.62 MHz
Stretch	5.8 MHz
xAl	3.5 MHz
xBe	13.0 MHz



CNOT gate implementation - 0

The initial state

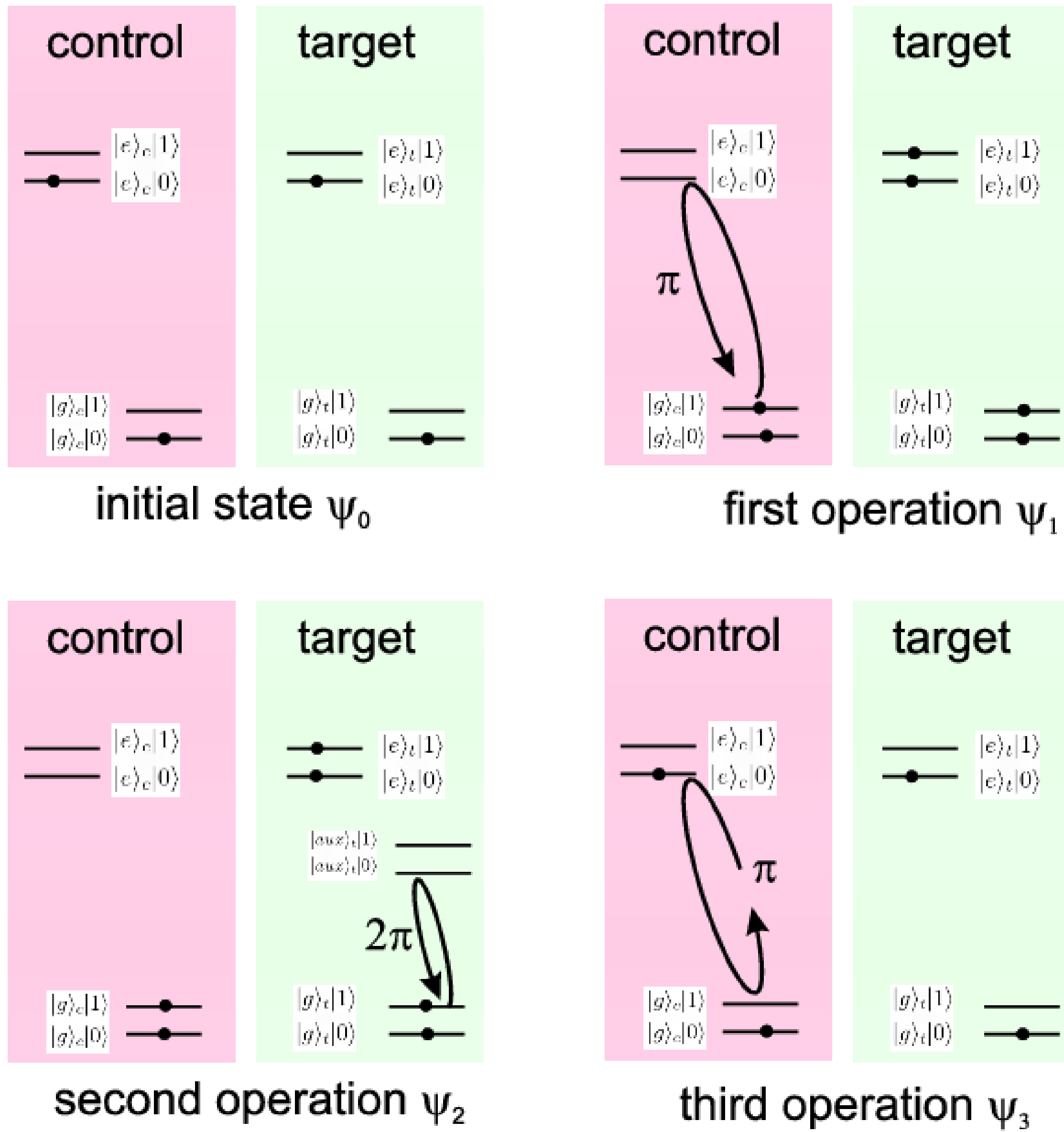
$$|g\rangle_c|g\rangle_t|0\rangle$$
$$|g\rangle_c|e\rangle_t|0\rangle$$
$$|e\rangle_c|g\rangle_t|0\rangle$$
$$|e\rangle_c|e\rangle_t|0\rangle$$



CNOT gate implementation – I

π-pulse on control ion
red detuned

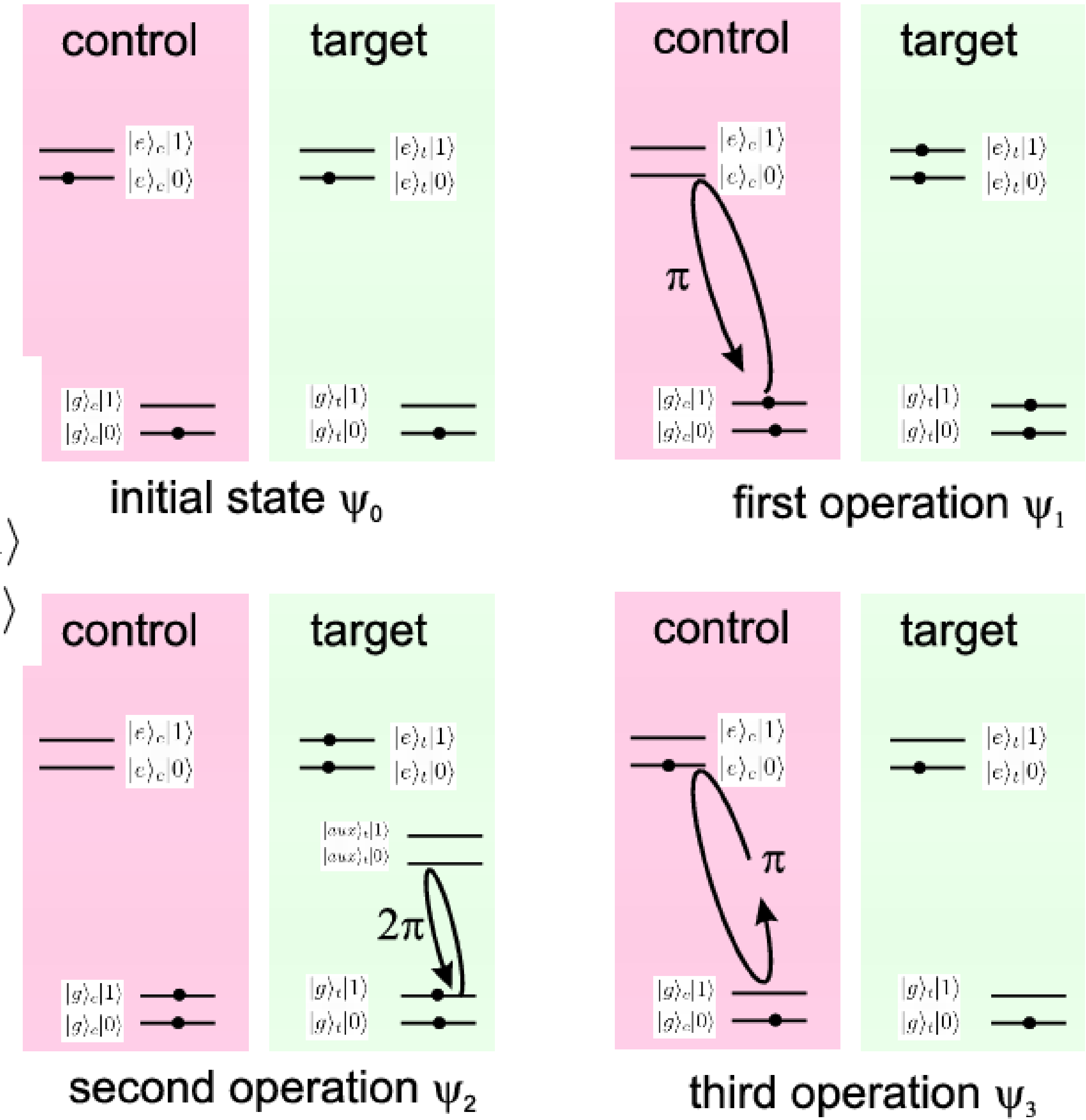
$$
\begin{aligned}
|g\rangle_c|g\rangle_t|0\rangle &\rightarrow |g\rangle_c|g\rangle_t|0\rangle \\
|g\rangle_c|e\rangle_t|0\rangle &\rightarrow |g\rangle_c|e\rangle_t|0\rangle \\
|e\rangle_c|g\rangle_t|0\rangle &\rightarrow -i|g\rangle_c|g\rangle_t|1\rangle \\
|e\rangle_c|e\rangle_t|0\rangle &\rightarrow -i|q\rangle_c|e\rangle_t|1\rangle
\end{aligned}
$$



CNOT gate implementation – II

2π-pulse on target ion auxiliary level

$$
\begin{aligned}
|g\rangle_c|g\rangle_t|0\rangle &\rightarrow |g\rangle_c|g\rangle_t|0\rangle \\
|g\rangle_c|e\rangle_t|0\rangle &\rightarrow |g\rangle_c|e\rangle_t|0\rangle \\
-i|g\rangle_c|g\rangle_t|1\rangle &\rightarrow +i|g\rangle_c|g\rangle_t|1\rangle \\
-i|g\rangle_c|e\rangle_t|1\rangle &\rightarrow -i|g\rangle_c|e\rangle_t|1\rangle
\end{aligned}
$$

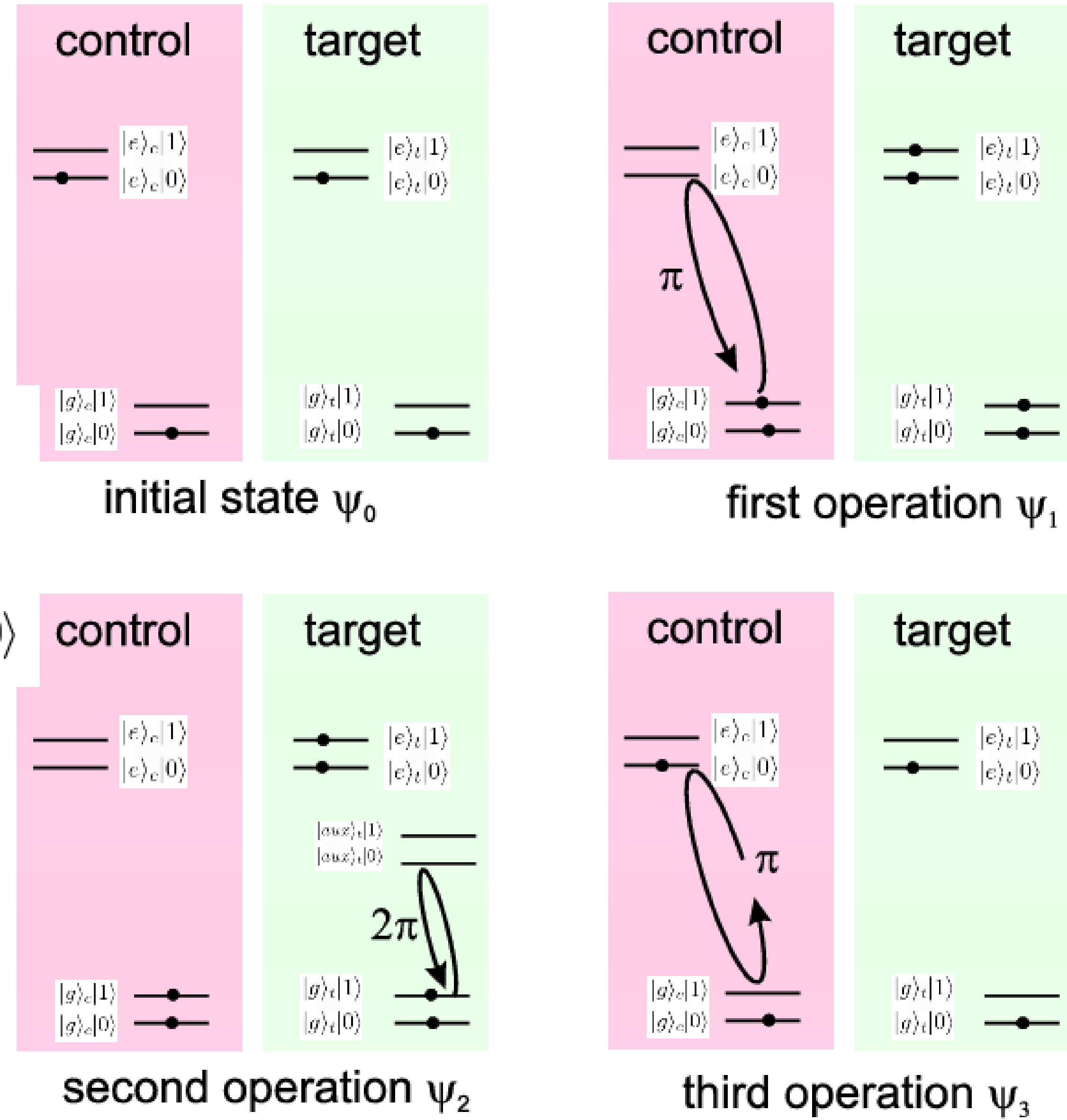


CNOT gate implementation – III

2π-pulse on target ion auxiliary level

$$
\begin{aligned}
|g\rangle_c|g\rangle_t|0\rangle &\rightarrow |g\rangle_c|g\rangle_t|0\rangle \\
|g\rangle_c|e\rangle_t|0\rangle &\rightarrow |g\rangle_c|e\rangle_t|0\rangle \\
+i|g\rangle_c|g\rangle_t|1\rangle &\rightarrow |e\rangle_c|g\rangle_t|0\rangle \\
-i|g\rangle_c|e\rangle_t|1\rangle &\rightarrow -|e\rangle_c|e\rangle_t|0\rangle
\end{aligned}
$$

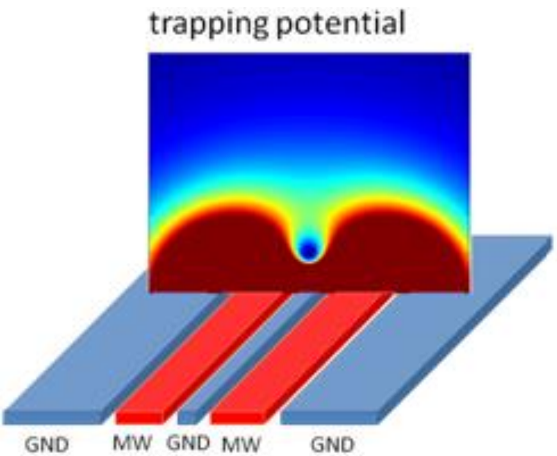
The phase gate!

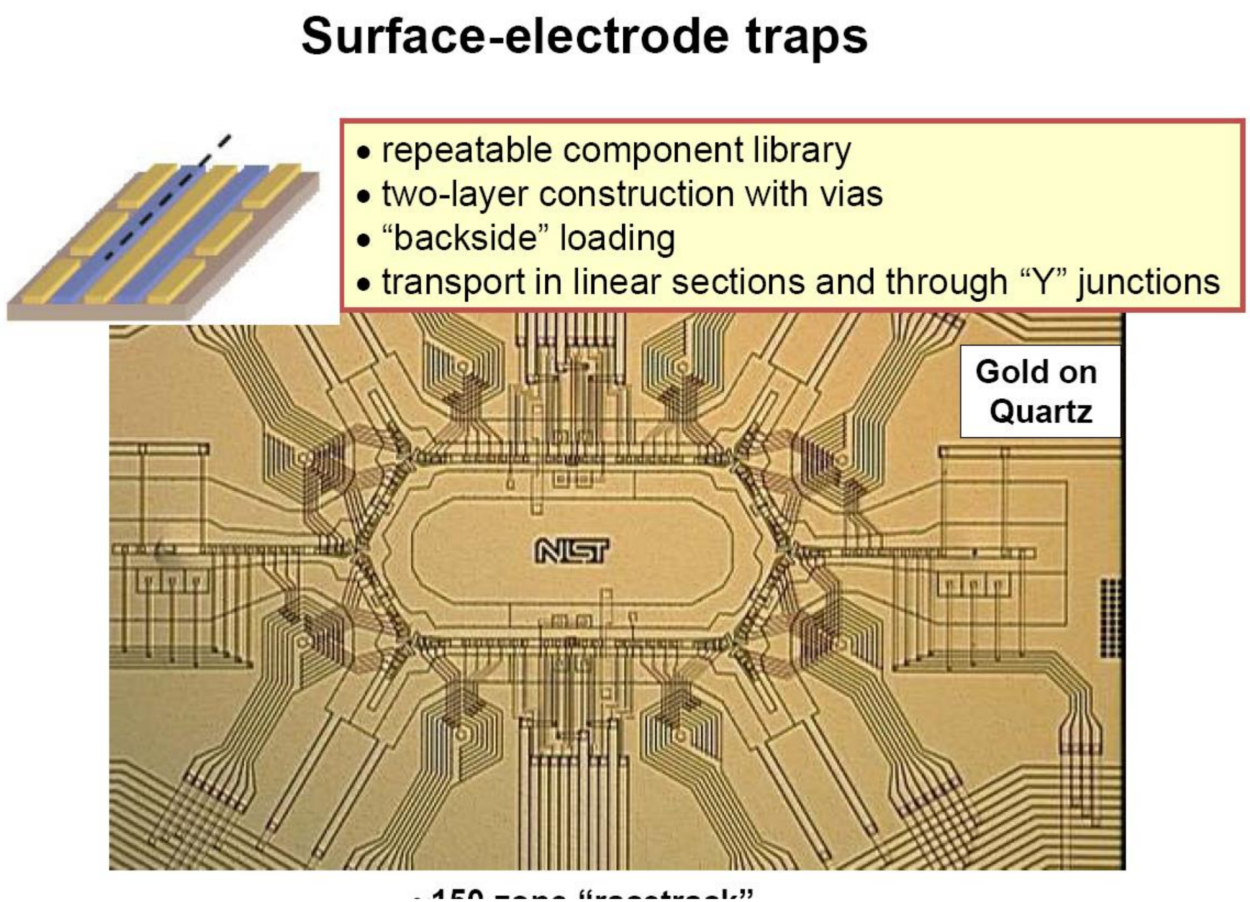


From the PHASE gate to the CNOT gate

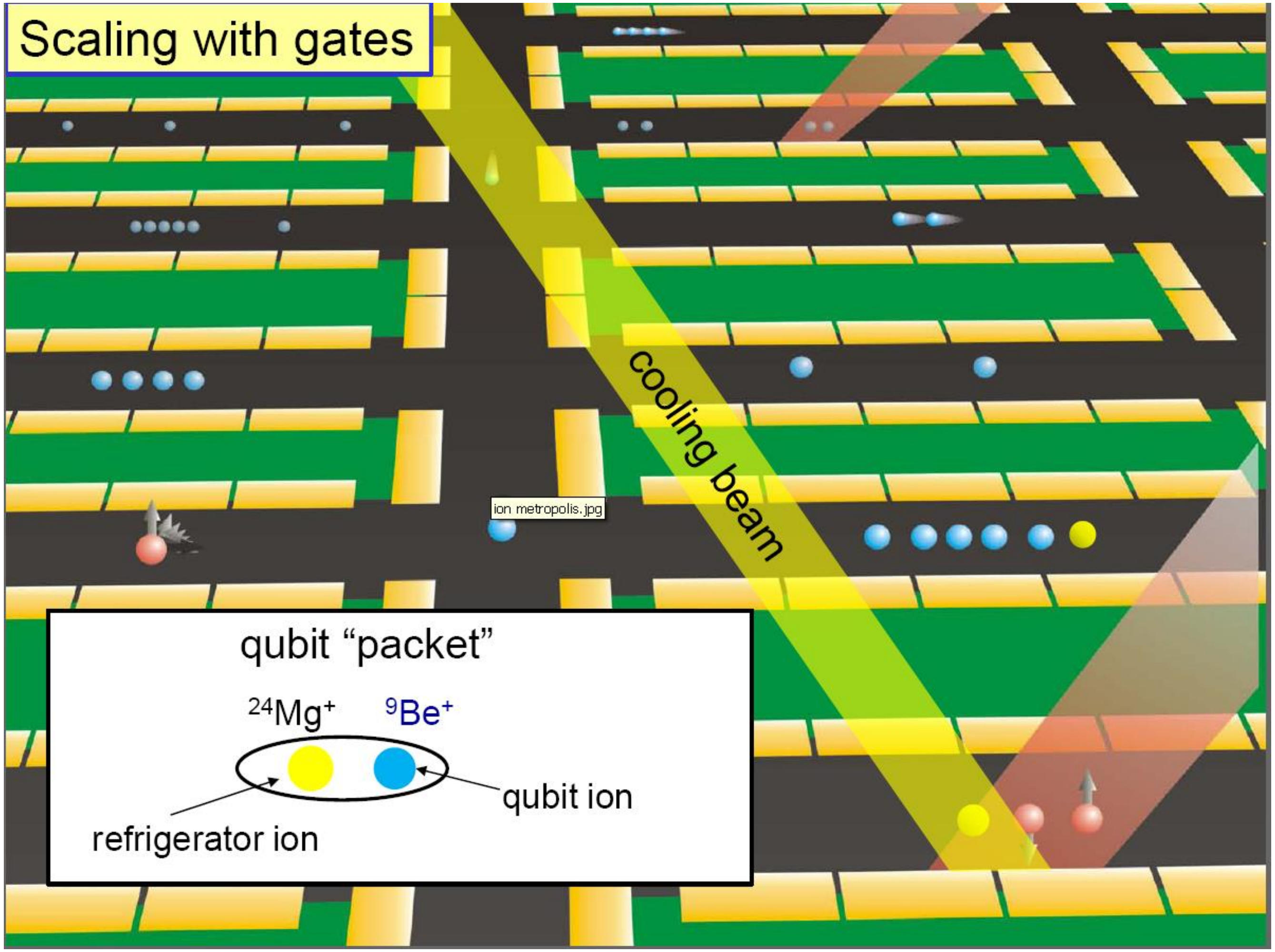
$$U_{\mathrm{CNOT}} = \frac{1}{\sqrt{2}}\begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{vmatrix} \cdot U_{\mathrm{phase}} \cdot \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{vmatrix} \frac{1}{\sqrt{2}}$$

Planar ion trap, approaching real computations





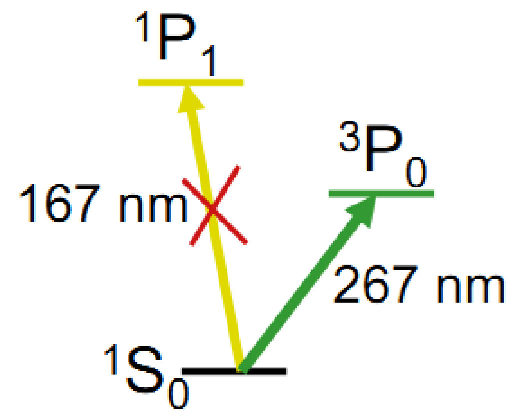
Symphatetic cooling by a sparring ion

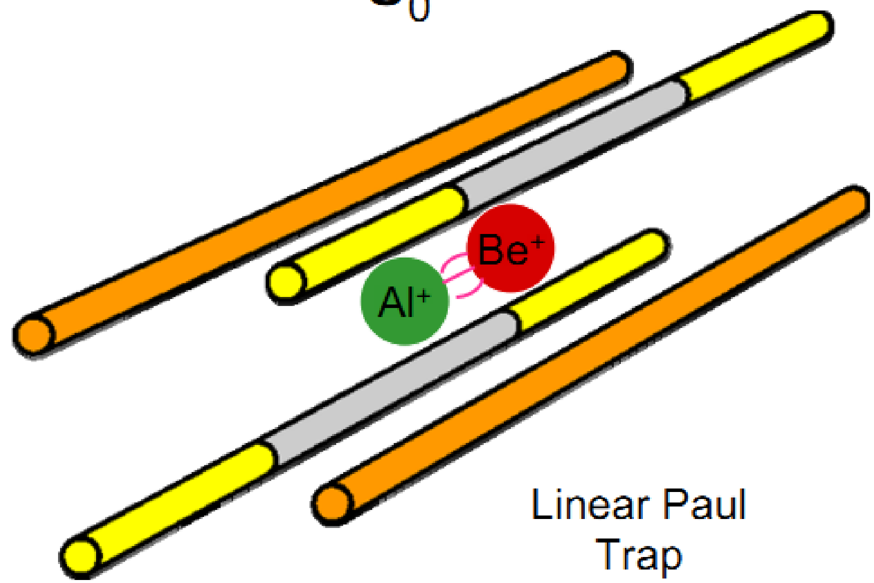


Реализован двухкубитный программируемый квантовый вычислитель с низким уровнем ошибок

Spectroscopy of $^{27}\mathrm{Al}^+$

- 8 mHz linewidth clock transition
- Insensitive to external fields
- Smallest known room temperature blackbody shift [2]
- No accessible strong transition for cooling & state detection
- Use two-ion quantum logic techniques with $^{9}\mathrm{Be}^+$ and $^{27}\mathrm{Al}^+$ for cooling, state preparation & readout [1]

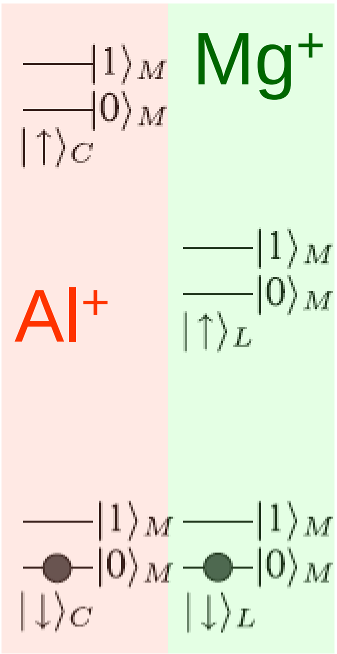




[1] D.J. Wineland *et al.*, Proc. 6th Symposium on Frequency Standards and Metrology, 2001, pp. 361-368

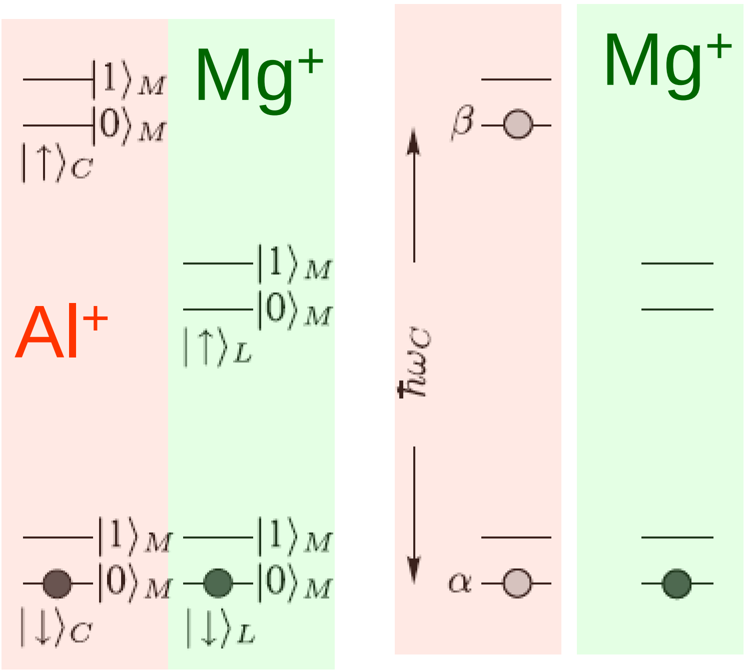
[2] T. Rosenband *et al.* arXiv:physics/0611125

Excitation transfer using the sparring ion



$$|\psi_0\rangle = |\downarrow\rangle_L\,|\downarrow\rangle_C\,|0\rangle_M$$

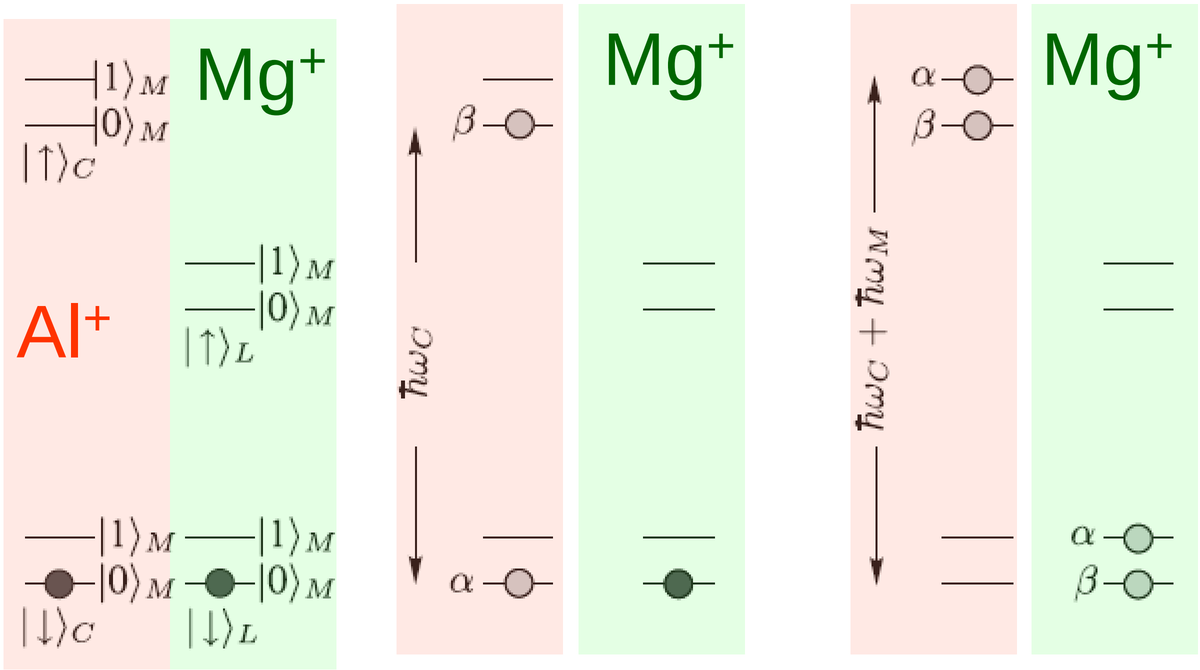
Excitation transfer using the sparring ion



$$|\psi_0\rangle = |\downarrow\rangle_L\,|\downarrow\rangle_C\,|0\rangle_M$$

$$\begin{aligned}|\psi_0\rangle \rightarrow |\psi_1\rangle &= |\downarrow\rangle_L\,[\alpha|\downarrow\rangle_C + \beta|\uparrow\rangle_C]\,|0\rangle_M = \\ &= |\downarrow\rangle_L\,[\alpha|\downarrow\rangle_C\,|0\rangle_M + \beta|\uparrow\rangle_C\,|0\rangle_M].\end{aligned}$$

Excitation transfer using the sparring ion

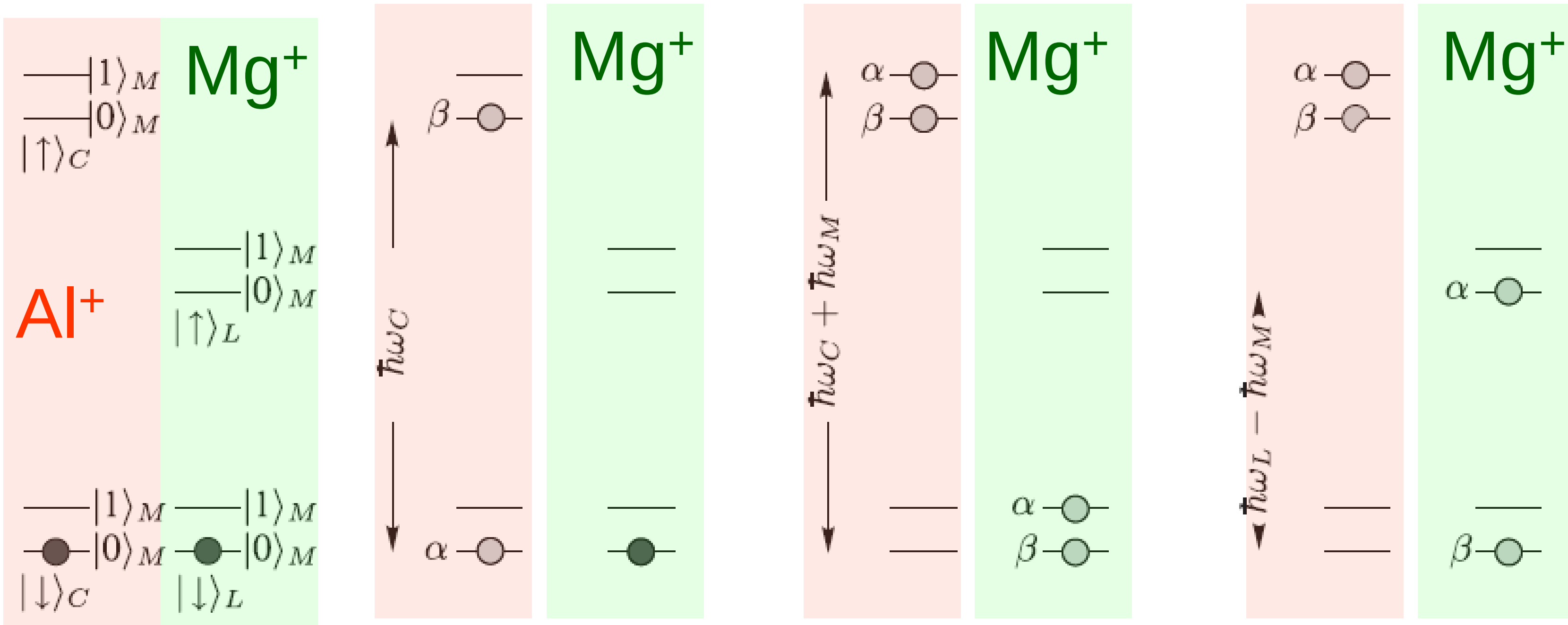


$$|\psi_0\rangle = |\downarrow\rangle_L\,|\downarrow\rangle_C\,|0\rangle_M$$

$$\begin{aligned}|\psi_0\rangle \to |\psi_1\rangle &= |\downarrow\rangle_L\,[\alpha|\downarrow\rangle_C + \beta|\uparrow\rangle_C]\,|0\rangle_M = \\ &= |\downarrow\rangle_L\,[\alpha|\downarrow\rangle_C\,|0\rangle_M + \beta|\uparrow\rangle_C\,|0\rangle_M].\end{aligned}$$

$$\begin{aligned}|\psi_1\rangle \to |\psi_2\rangle &= |\uparrow\rangle_L\,[\alpha|\uparrow\rangle_C\,|1\rangle_M + \beta|\uparrow\rangle_C\,|0\rangle_M] = \\ &= |\downarrow\rangle_L\,|\uparrow\rangle_C\,[\alpha\,|1\rangle_M + \beta\,|0\rangle_M].\end{aligned}$$

Excitation transfer using the sparring ion



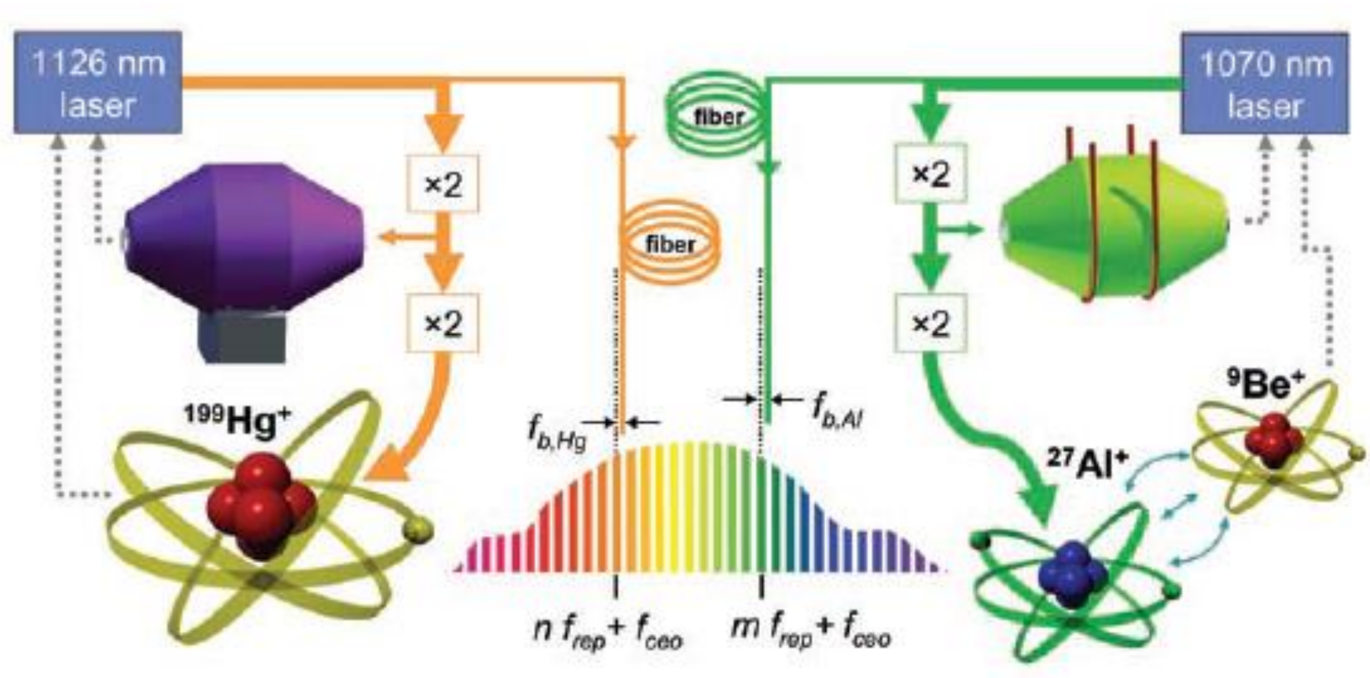
$$|\psi_0\rangle = |\downarrow\rangle_L\,|\downarrow\rangle_C\,|0\rangle_M$$

$$\begin{aligned}|\psi_0\rangle \to |\psi_1\rangle &= |\downarrow\rangle_L\,[\alpha|\downarrow\rangle_C + \beta|\uparrow\rangle_C]\,|0\rangle_M = \\ &= |\downarrow\rangle_L\,[\alpha|\downarrow\rangle_C\,|0\rangle_M + \beta|\uparrow\rangle_C\,|0\rangle_M].\end{aligned}$$

$$\begin{aligned}|\psi_1\rangle \to |\psi_2\rangle &= |\uparrow\rangle_L\,[\alpha|\uparrow\rangle_C\,|1\rangle_M + \beta|\uparrow\rangle_C\,|0\rangle_M] = \\ &= |\downarrow\rangle_L\,|\uparrow\rangle_C\,[\alpha\,|1\rangle_M + \beta\,|0\rangle_M].\end{aligned}$$

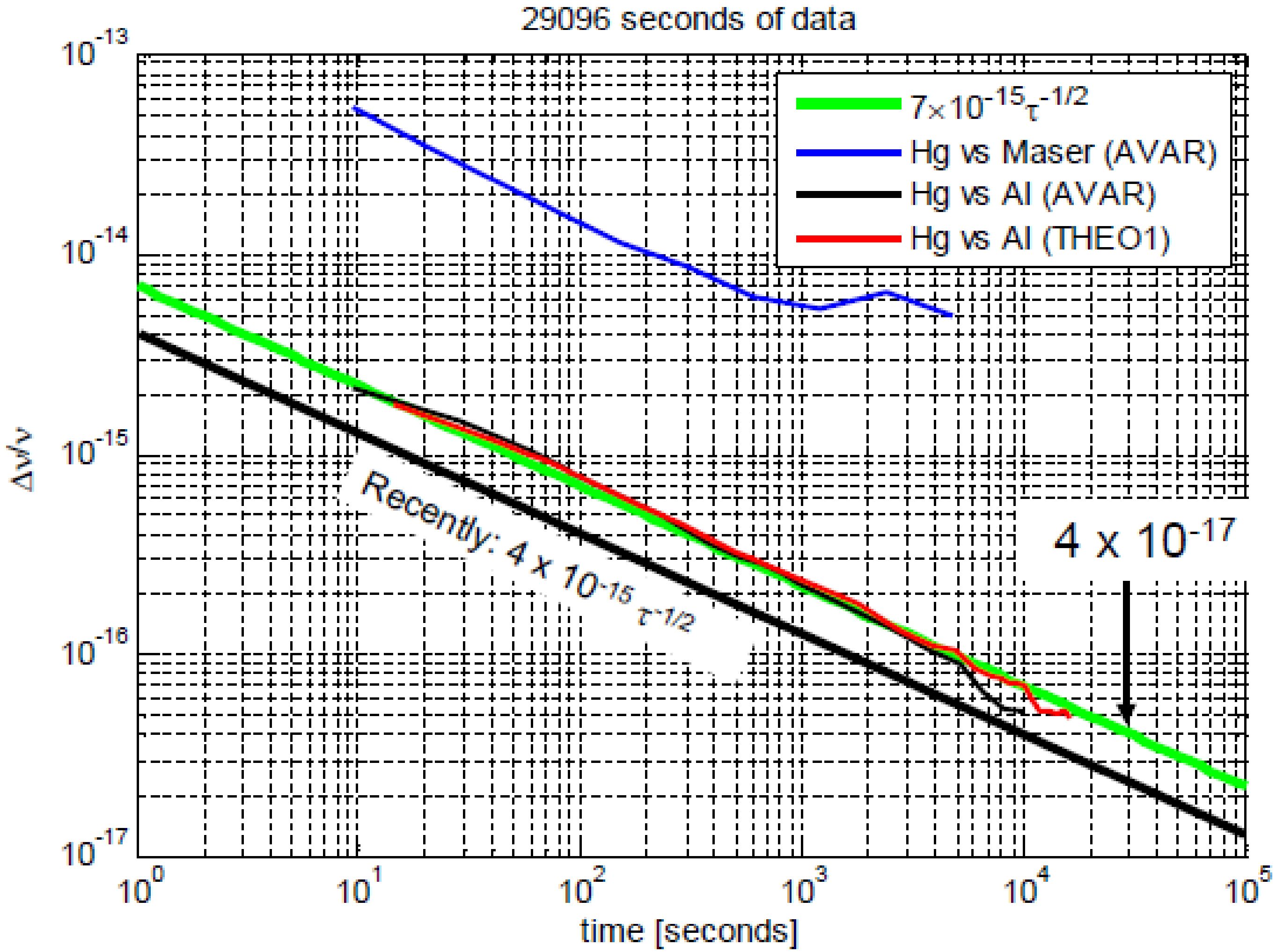
$$|\psi_2\rangle \to |\psi_{\text{final}}\rangle = [\alpha\,|\uparrow\rangle_L + \beta\,|\downarrow\rangle_L]\,|\uparrow\rangle_C\,|0\rangle_M.$$

Frequency Ratio of Al^+ and Hg^+ Single-Ion Optical Clocks; Metrology at the 17th Decimal Place



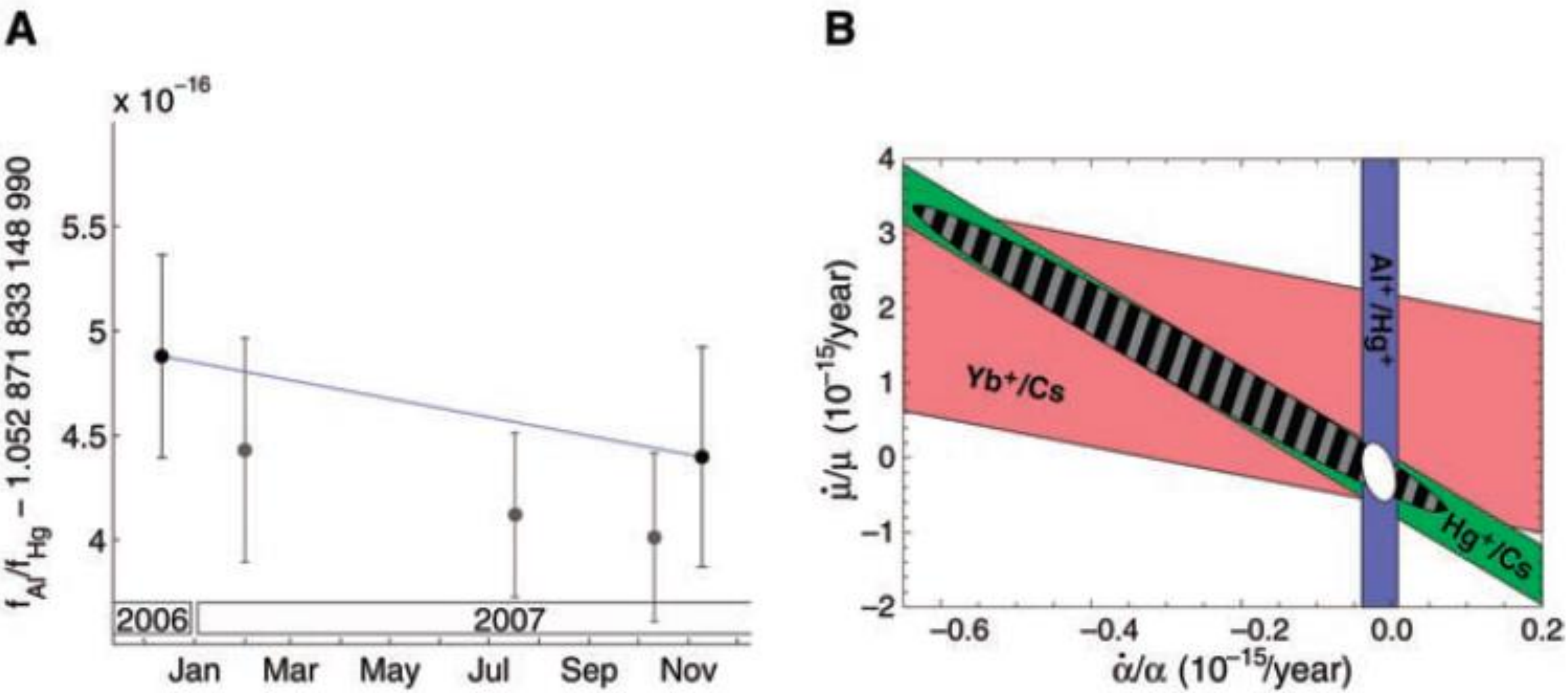
Science (2008)

Al^+/Hg^+ Stability



Science (2008)

Most stringent restriction for the variation of the fine structure constant



$$\dot{\alpha}/\alpha = (-1.6 \pm 2.3) \times 10^{-17}/\text{year}.$$

Science (2008)

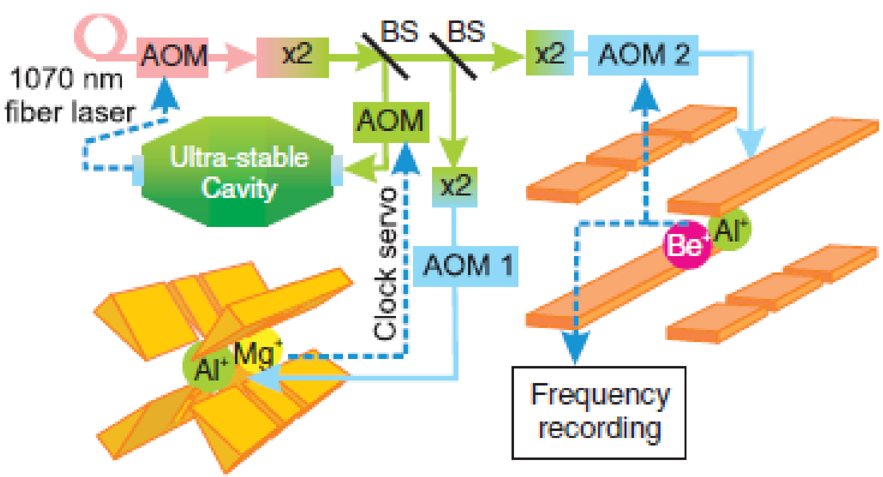
Frequency Comparison of Two High-Accuracy Al^+ Optical Clocks

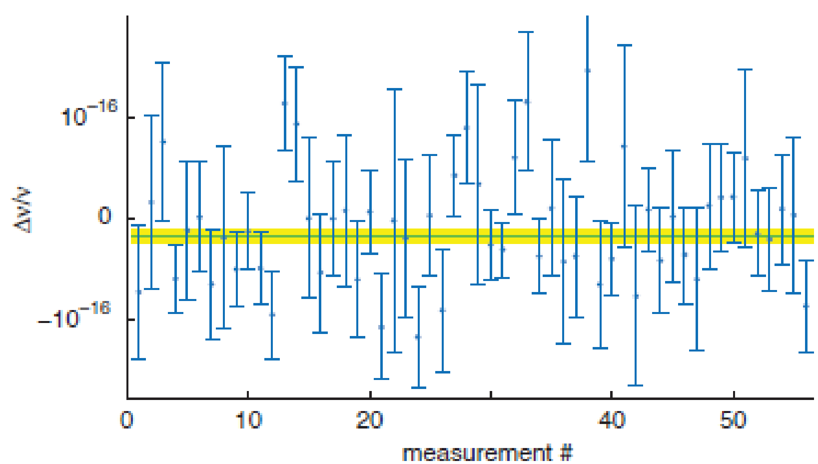
C. W. Chou,* D. B. Hume, J. C. J. Koelemeij,† D. J. Wineland, and T. Rosenband

Time and Frequency Division, National Institute of Standards and Technology, Boulder, Colorado 80305, USA

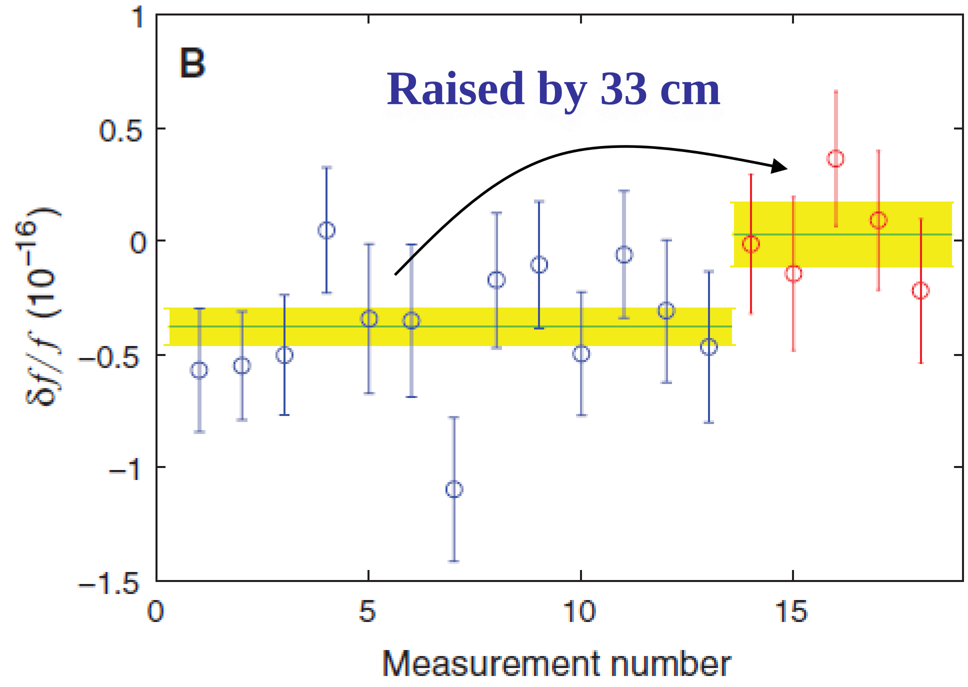
(Received 23 November 2009; published 17 February 2010)

We have constructed an optical clock with a fractional frequency inaccuracy of 8.6×10^{-18}, based on quantum logic spectroscopy of an Al^+ ion. A simultaneously trapped Mg^+ ion serves to sympathetically laser cool the Al^+ ion and detect its quantum state. The frequency of the ${}^1S_0 \leftrightarrow {}^3P_0$ clock transition is compared to that of a previously constructed Al^+ optical clock with a statistical measurement uncertainty of 7.0×10^{-18}. The two clocks exhibit a relative stability of $2.8 \times 10^{-15}\tau^{-1/2}$, and a fractional frequency difference of -1.8×10^{-17}, consistent with the accuracy limit of the older clock.





Gravitational red shift observation in the lab



ФИАН

Nobel Prize in Physics, 2012

David Wieland

NIST, USA