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Lecture 5

- Global navigation system structure – space segment, ground segment, user segment. Satellites orbits, frequency shifts, accuracy.
- Data coding and decoding. CDMA, method.
- Atmospheric errors, corrections, clock synchronization.

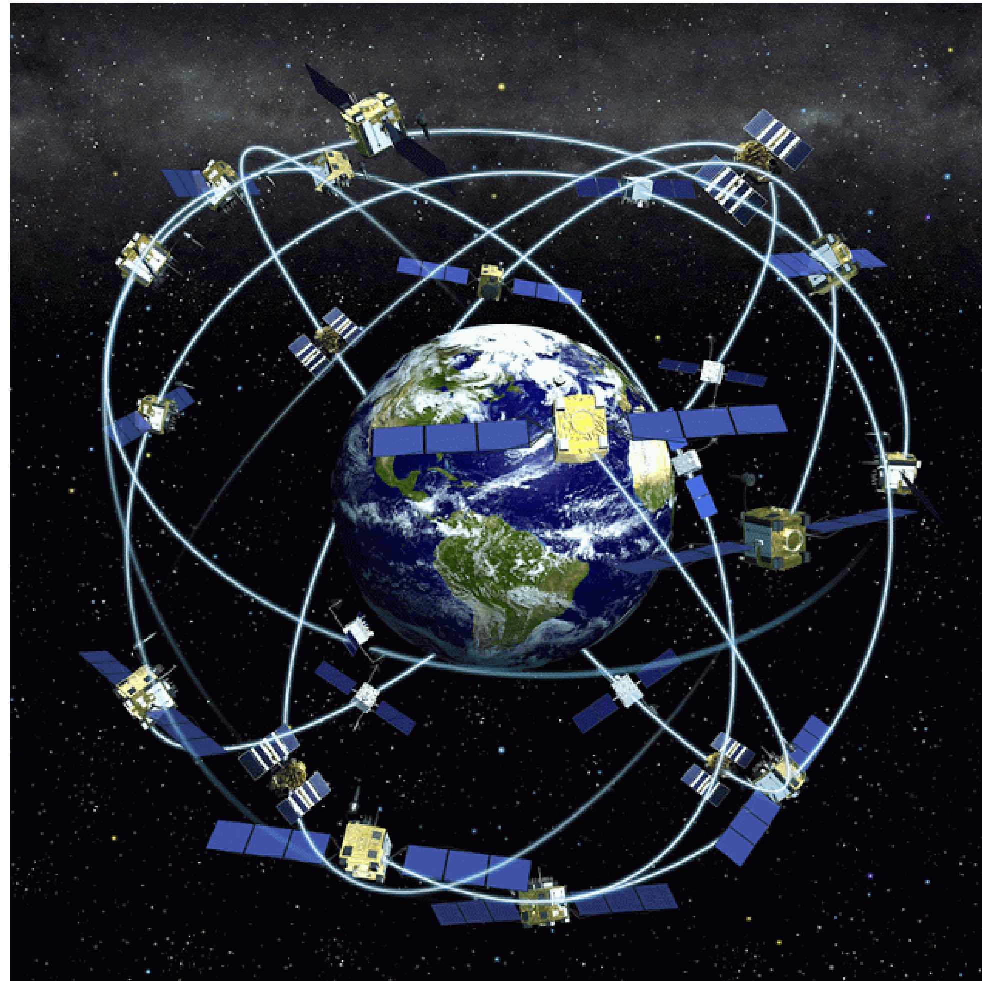
Global satellite navigation system

USA : GPS

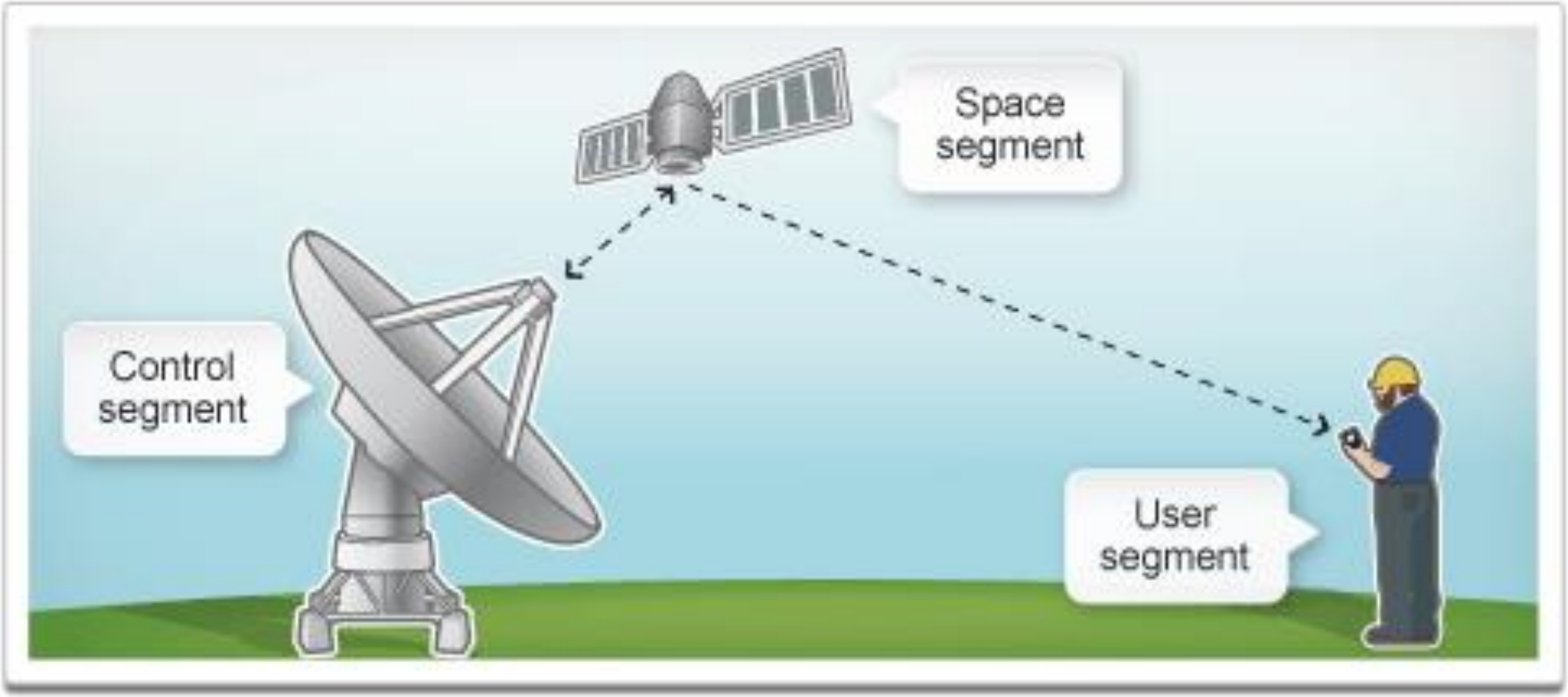
Russia: GLONASS

Europe: GALILEO

China: COMPASS

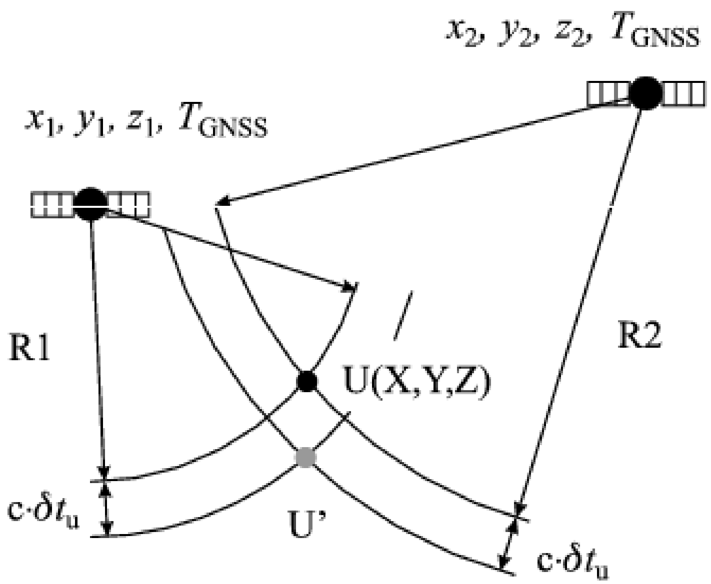


GPS segments



Ground stations

Calculation of positions



$$R_1 = c \cdot \Delta t_1$$

If the error of time synchronization is only

$$\delta t = 1\ \mu s$$

The error in position determination will be

$$300\ \text{m}$$

User clock

$$\delta t_u = T_U - T_{GNSS}$$

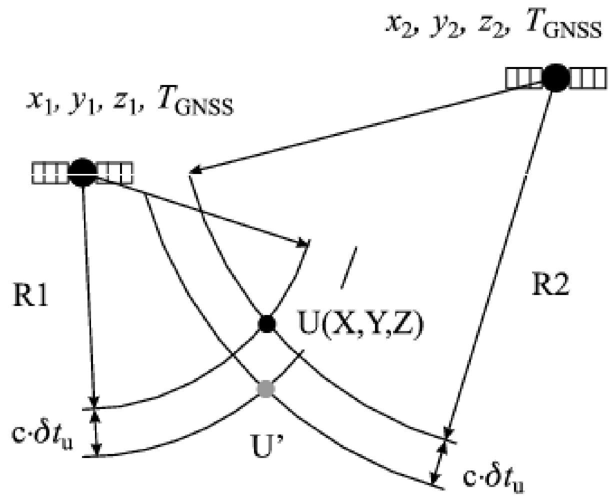
GPS clock

pseudo-distance

$$P_i = R_i + c \cdot \delta t_u$$

Non-linear equations

$$P_i = R_i + c \cdot \delta t_u$$



$$
\begin{aligned}
(x_1 - X)^2 + (y_1 - Y)^2 + (z_1 - Z)^2 &= (P_1 - c\,\delta t_u)^2\,,\\
(x_2 - X)^2 + (y_2 - Y)^2 + (z_2 - Z)^2 &= (P_2 - c\,\delta t_u)^2\,,\\
(x_3 - X)^2 + (y_3 - Y)^2 + (z_3 - Z)^2 &= (P_3 - c\,\delta t_u)^2\,,\\
(x_4 - X)^2 + (y_4 - Y)^2 + (z_4 - Z)^2 &= (P_4 - c\,\delta t_u)^2\,.
\end{aligned}
$$

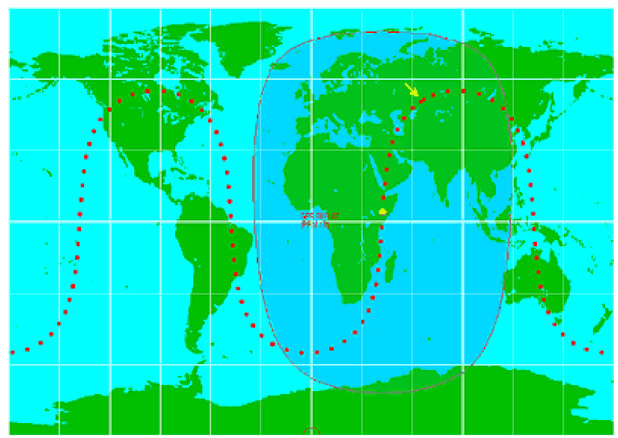
Satellite orbits

$$G\frac{M_E M_S}{R^2} = M_S \omega^2 R$$

$$G M_E = 3,986\,004\,418 \cdot 10^{14}\ \mathrm{m}^3/\mathrm{s}^2$$

Orbit period: **12 hours - 2 min** (simplifies calculations)

Orbit radius: **26 560 km**



Orbit eccentricity: **0.02**

Min number of satellites: **24**

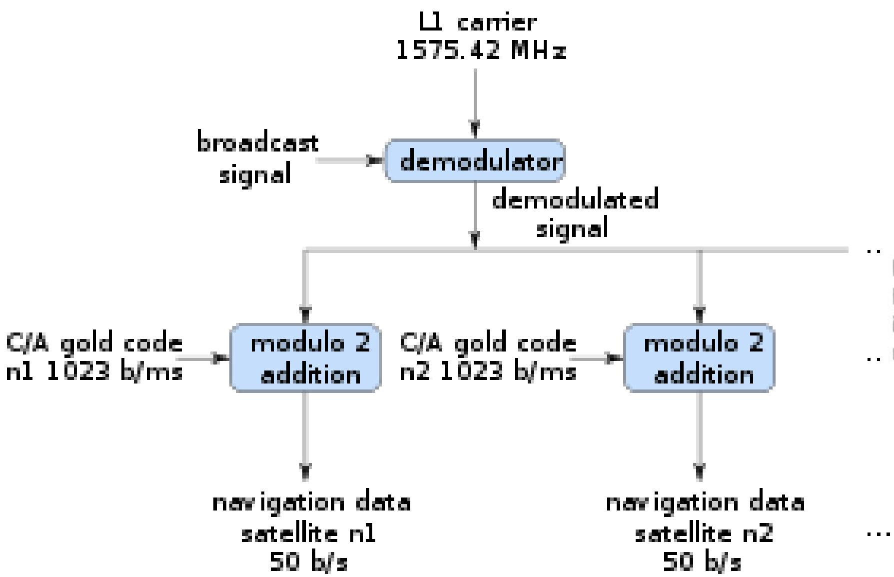
On-board clocks

Cs/Rb atomic clocks (GPS)

H-maser (Galileo)

Disseminated frequencies

Above 2 GHz – beam antenna necessary
Below 100 MHz – ionospheric delays are huge!
Needs high bandwidth to transmit PRN codes



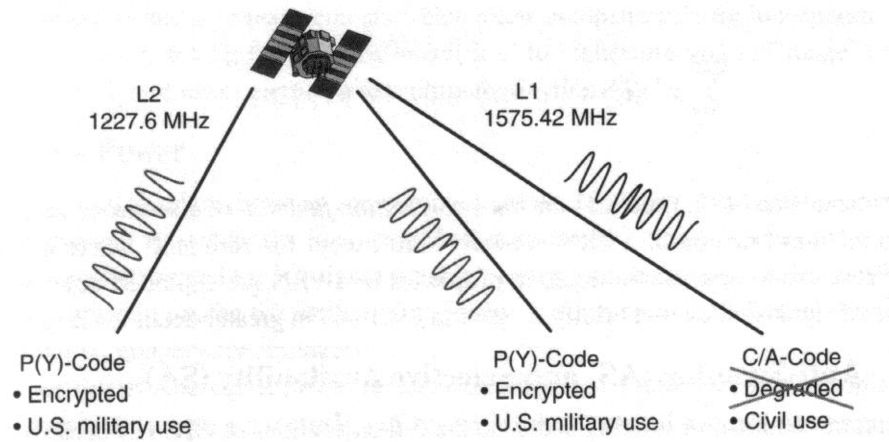
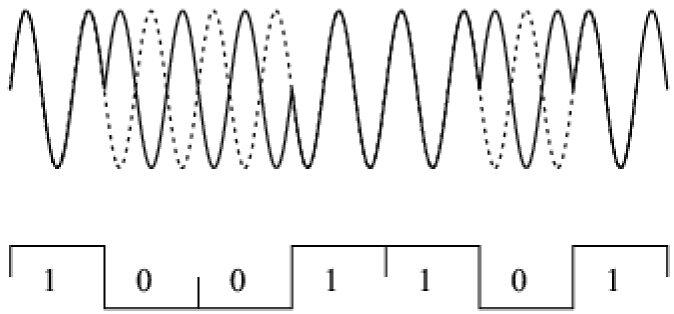


Figure 2.4 GPS signals. Currently, each GPS satellite transmits three signals, two on L1 and one on L2 frequency. The BPSK-modulated signals are shown. The signal carrying C/A-code on L1 was degraded purposely throughout the 1990s, but this practice has now ended. Access to P(Y)-code is limited to the DoD-authorized users via encryption.

ФИАН

Data coding



Carrier

Pseudo-random code

Figure 5.3: *Phase modulation by a pseudo-random code (PRN).*

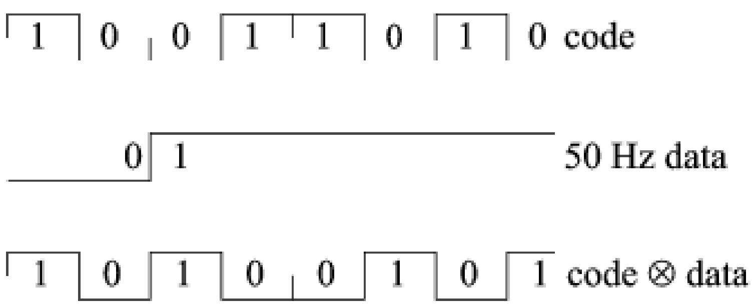
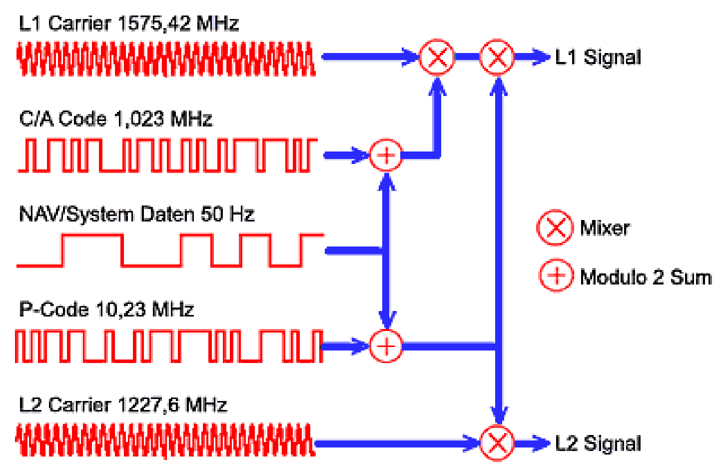
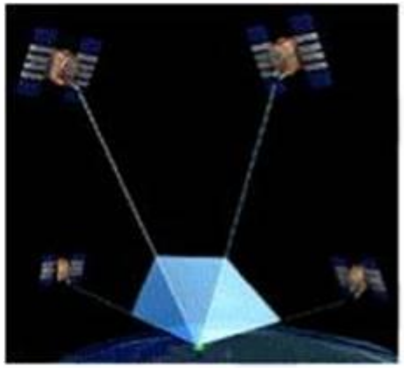
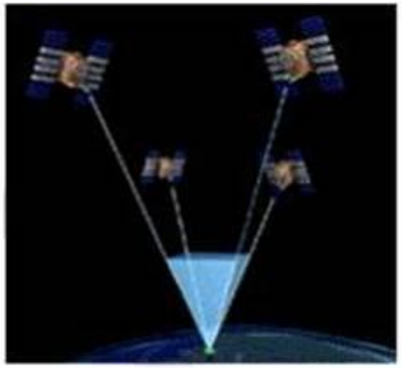


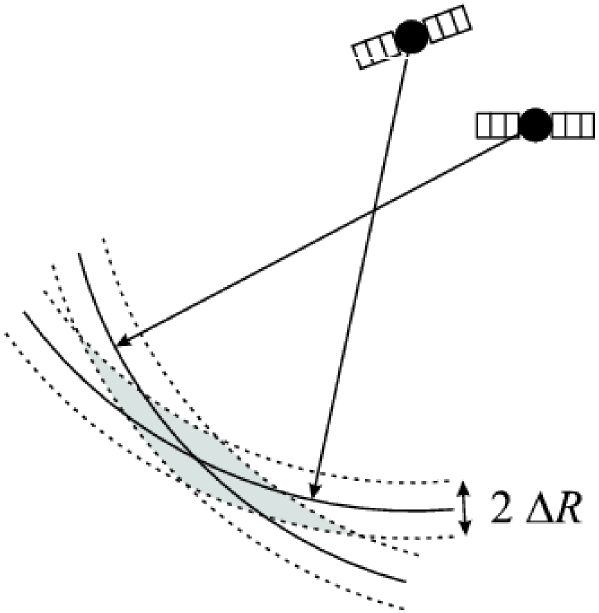
Figure 5.4: *Data coding in GPS signal.*



Uncertainties in GPS system: I







Geometric dilution of precision GDOP

Uncertainties in GPS system: II

Effects of General Relativity

$$U = -\frac{GM_E}{R} - \frac{\omega^2 R^2}{2}\,. \tag{5.3}$$

For the clock on board of the satellite we get

$$U_{\text{satellite}} = -\frac{GM_E}{R} - \frac{GM_E}{2R} = -\frac{3}{2}\frac{GM_E}{R}\,, \tag{5.4}$$

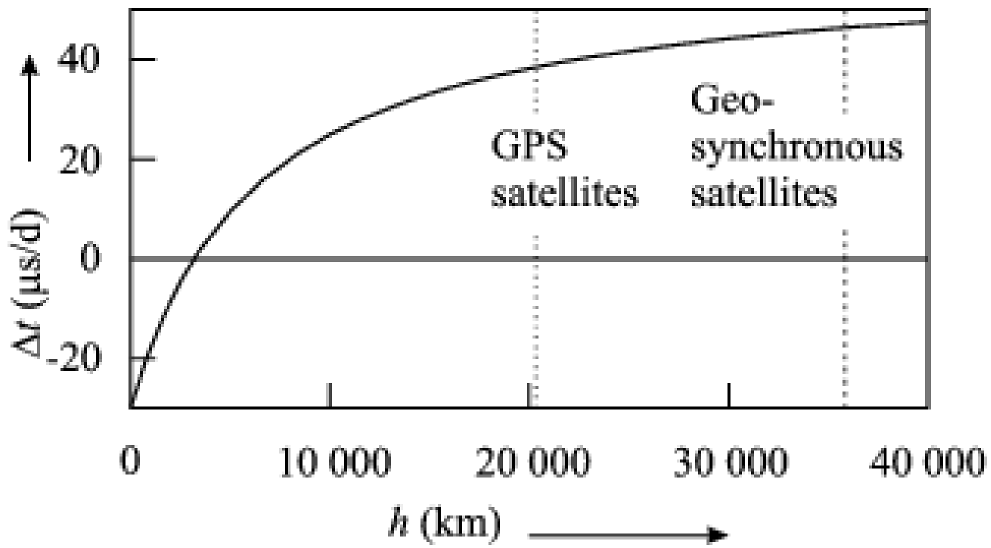
using eqs. (5.3) (5.2).

For the clock resting on the geoid's surface we get $U_{\text{surface}} = -62,6\ (\text{km/s})^2$. The potential difference between two clocks result in the time difference of

$$\frac{\Delta\nu}{\nu} = \frac{\Delta U}{c^2} = \frac{1}{c^2}\left(-\frac{3}{2}\frac{GM_E}{R} + 62,6\cdot 10^6\ \frac{\text{m}^2}{\text{s}^2}\right)\,. \tag{5.5}$$

Uncertainties in GPS system: II

Effects of General Relativity



Transmitted

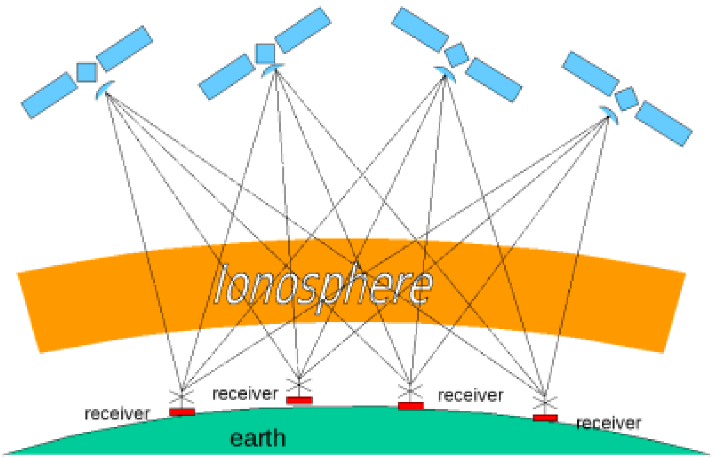
10,229 999 995 432 6 MHz

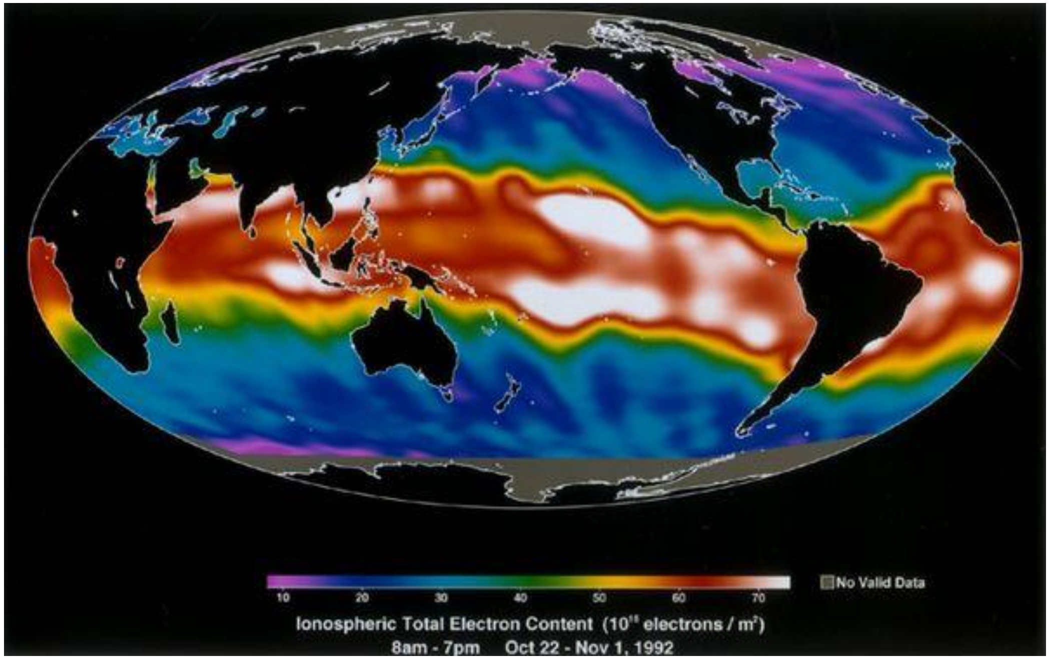
Received

10,23 MHz

Uncertainties in GPS system: III

Ionospheric delays





Index of refraction (phase velocity)

$$n_p = 1 + \frac{c_2}{\nu^2}$$

$$c_2 = -40.3 \times n_e \ \mathrm{Hz}^2$$

Uncertainties in GPS system: III

Index of refraction (phase velocity)

$$n_p = 1 + \frac{c_2}{\nu^2}$$

$$c_2 = -40.3 \times n_e \ \mathrm{Hz}^2$$

Group velocity

$$c/n_g$$

$$n_g = n_p + \nu dn_p/d\nu$$

We get: $$n_g = 1 - \frac{c_2}{\nu^2}$$

Thus, the ionospheric delay for the data transfer can be given by

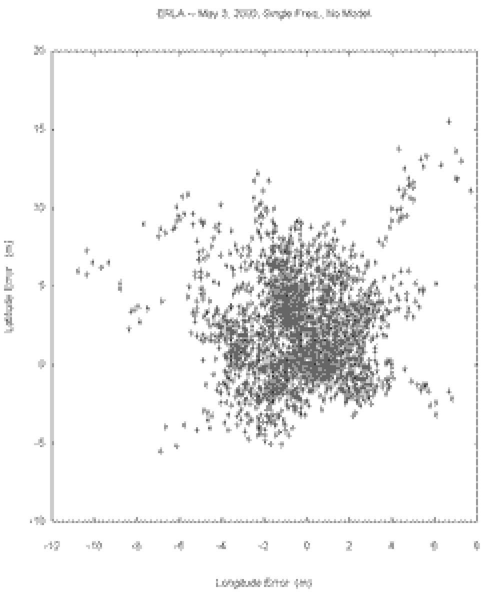
$$\Delta T = \frac{40,3 \cdot \mathrm{TEC}}{c\nu^2}.$$

One can measure
ionospheric delay

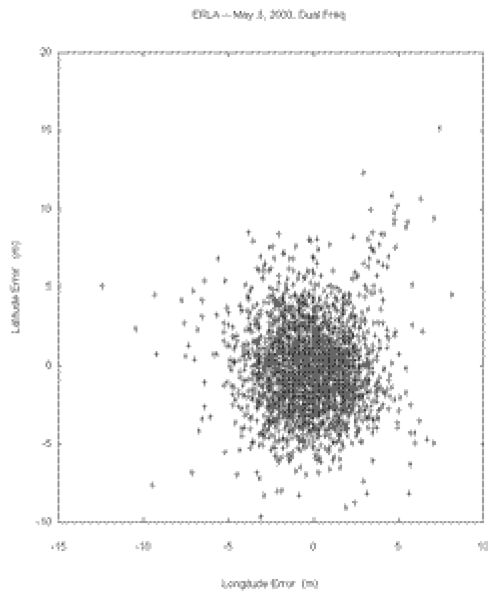
$$\Delta\tilde{T} \equiv \Delta T(\mathrm{L1}) - \Delta T(\mathrm{L2}) = \frac{40,3 \cdot \mathrm{TEC}}{c}\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right) = \Delta T(\mathrm{L1})\frac{\nu_2^2 - \nu_1^2}{\nu_2^2}$$

Uncertainties in GPS system: III

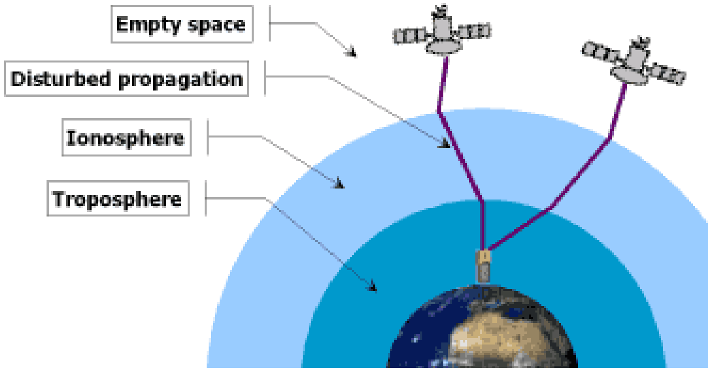
Without correction



With correction

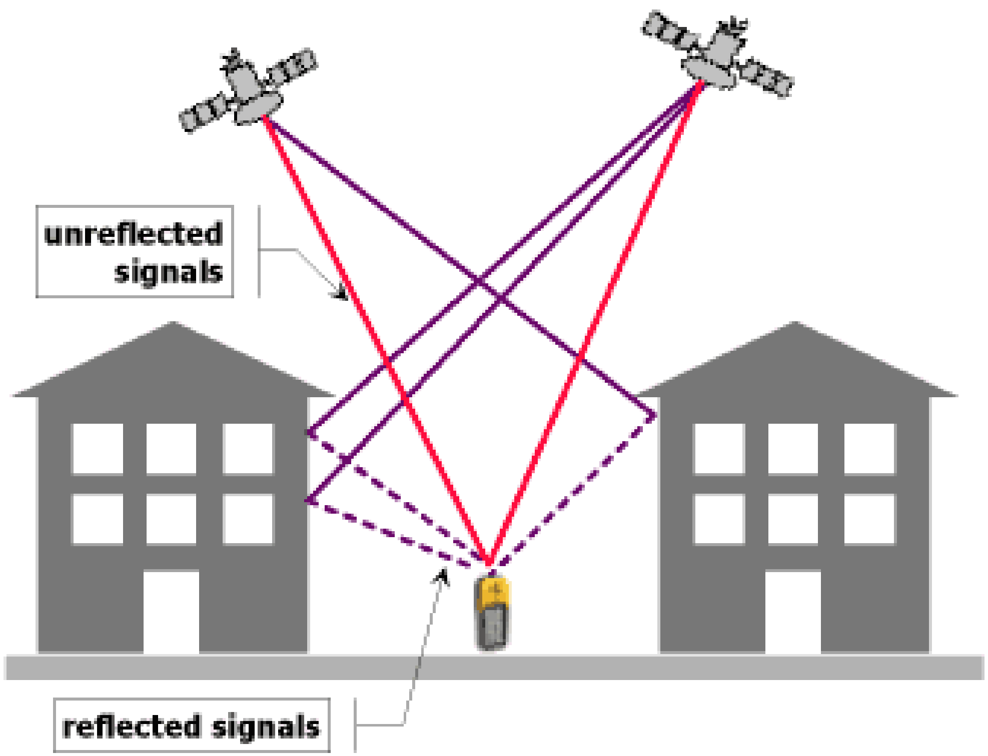


Tropospheric delays cannot be corrected (no dispersion)



Technical uncertainties:

Multipath propagation

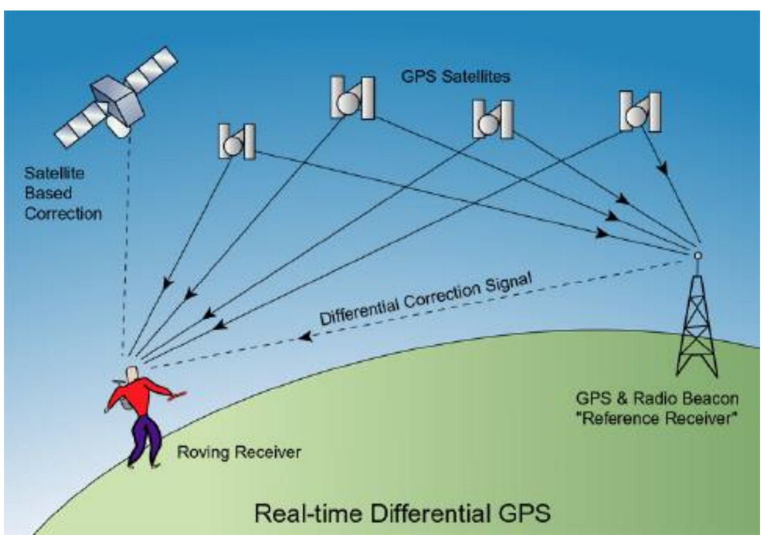


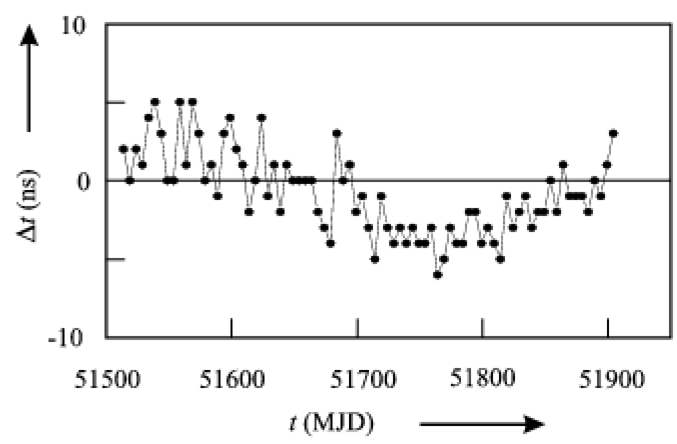
Total uncertainty budget

Source of uncertainty	uncertainty
on-board clocks	3,0 m
satellite orbits	1,0 m
other perturbations	0,5 m
ephemerides prediction	4,2 m
other	0,9 m
ionospheric delay	2,3 m
tropospheric delay	2,0 m
receiver nose	1,5 m
propagation	1,2 m
by different channels	
others	0,5 m
sum	6,6 m

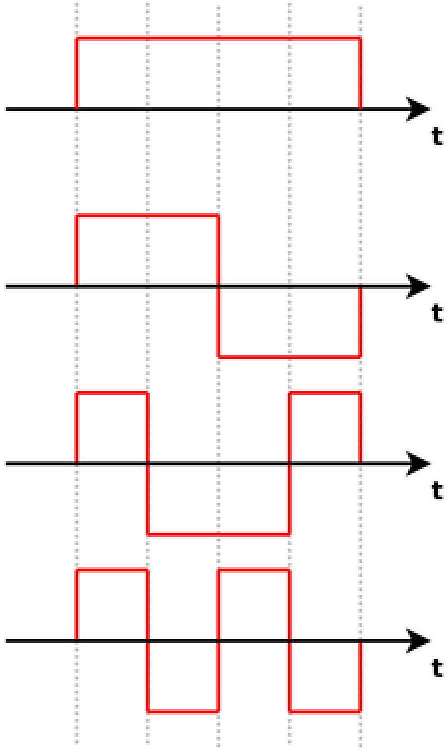
Different acquisition methods

method	time uncertainty	relative frequencys uncertainty
one-way	$< 20\,\text{ns}$	$< 2 \cdot 10^{-13}$
one-channel differential	$\approx 10\,\text{ns}$	$\approx 10^{-13}$
multi-channel differential	$< 5\,\text{ns}$	$< 5 \cdot 10^{-14}$
differential with carrier phase measurement	$< 500\,\text{ps}$	$< 5 \cdot 10^{-15}$





CDMA basics



vectors

$$u = (a, b) \quad v = (c, d)$$

product

$$u \cdot v = ac + bd$$

Orthogonal set : Walsh matrices

$$H(2^1) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$

$$H(2^2) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix},$$

and in general

$$H(2^k) = \begin{bmatrix} H(2^{k-1}) & H(2^{k-1}) \\ H(2^{k-1}) & -H(2^{k-1}) \end{bmatrix} = H(2) \otimes H(2^{k-1}).$$